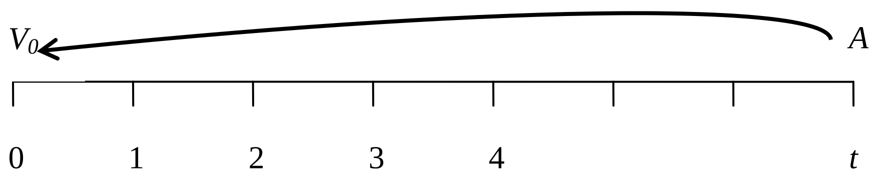
Discounting Basic Equations

Discount rate: annual or periodic rate at which future benefits or costs are discounted relative to the current benefits or costs. (Based on the idea that humans are impatient and that production takes time; see Economics of Time.)

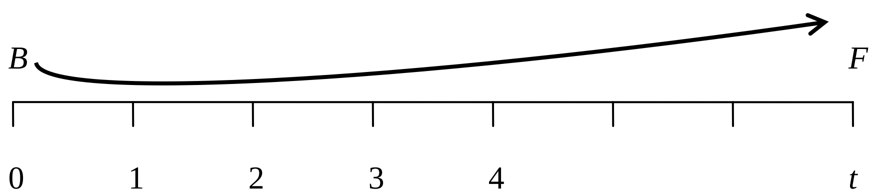
Simple discount: $V_0 = \dfrac{A}{(1+i)^t}$

V= Present Value, *A*= A single future payment, i = interest rate or discount rate,
t = number of time periods usually measured in years
Time Line:



Future Value of Investments; compound interest: If you invest *B* dollars today, at time 0, your investment will be worth at time n:

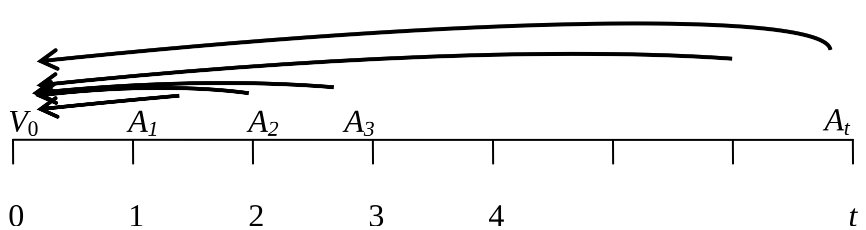
$$F_n = B(1+i)^t$$



Complex discount:

$$V_0 = \sum_{1}^{t} \frac{A_n}{(1+i)^n}$$

A_n are payments at time n, starting at time 1—not time 0—and ending at time t. Note that the payments may all be *different*.



Level Payment: all the payments from time 1 to time *n* are identical. This is the formula for mortgages, student loans, car and other time purchases, credit card minimum payments etc.

$$V_0 = \sum_1^t \frac{a}{(1+i)^n} = a\frac{1-\frac{1}{(1+i)^t}}{i}$$ where *a* is the *identical* payment in each period, starting at time 1—not time 0—and ending at time *t*. ("*a*" stands for "annuity".)

Infinite Level Payment—Value of Land and other Durable Assets

If *ti*>>1, we can drop the $\frac{1}{(1+i)^t}$ term and write the present value simply:

$$V_0 = \frac{a}{i}$$

Because land lasts forever, we use this formula for **land value**. However, it applies to any reasonably durable asset that yields a level flow of returns, for example, a dam or a skyscraper. In the case of land or other long-lived natural capital, *a* is economic rent.

Land Value with growth and taxes:

If we expect the returns of land (or other durable asset) to increase by a certain rate $g < i$ per period, the formula becomes:

$$V_0 = \frac{a}{i-g}$$ **,** which is greater than $V_0 = \frac{a}{i}$

That is, expectations of growth increase land values, dramatically if *g* is close to *i*. (*g* must however be $<i$ or the system blows up.)

On the other hand, a tax at rate *r* on the value of land decreases that value:

$$V_0 = \frac{a}{i+r}$$ **,** which is less than $V_0 = \frac{a}{i}$

Combining taxes with expectations of growth:

$$V_0 = \frac{a}{i-g+r}$$

Thus taxes can counteract expectations of growth, shrinking real estate bubbles