CHAPTER 8

DYNAMIC EQUILIBRIUM DISTRIBUTION IN A SIMPLE ECONOMY

In the real world, distributions of wealth have other salient characteristics besides stability:

1. Save in the most primitive societies, distribution is and has been throughout the history of civilization, relentlessly unequal. Even after massive redistribution, as in communist revolutions, inequality seems to reassert itself.

2. The upper "tails" of distributions are far too long for any plausible random process to account for them. That is, the rich are far too rich to explain by luck.

3. Where distribution is extremely unequal, as in all but the very primitive less developed countries, it takes a characteristic "dual" form. A few "oligarchs" occupy the top of the social scale, and a large poverty-stricken mass occupies the bottom, with virtually no middle class in between. Many observers find a tendency to dualism in developed countries, and even among different size firms! For example, Robert Averitt describes American industry as The Dual Economy [Averitt, 1968].

4. Economic growth in most less developed countries makes distribution yet more unequal. In developed countries, or at least in the U.S and Great Britain where the evidence is clearest, growth does not seem to worsen inequality.

If transactions costs create decreasing returns to scale and if, as hypothesized in Chp. 4, future-orientation increases with wealth, a very simple further hypothesis can explain the above observations: At small wealth, decreasing returns to scale dominate; while at large wealth, increasing future-orientation dominates.

8.1 Results and Further Implications of Chp. 8 Models[A]

Equal and Dual Distribution in a Simple Economy:

Return to the multi-period Clone economy of Chp. 4, where a number of identical self-sufficient farmers occupy identical quality land which they can freely buy and sell at a market price. The hypothesis that decreasing returns dominate at small wealth, while increasing future-orientation dominates at large wealth has these consequences:

For a small enough area of land per capita and small enough number of farmers, only equal distribution is a stable dynamic equilibrium position. If individuals are displaced up and down from the equal distribution land size, the ones with more land sell to those with less--so everyone gradually returns to equal distribution.

Suppose land area per capita and/or number of farmers increases. As long as land per capita stays below a critical value, equal distribution remains a position of stable dynamic equilibrium. However, a new position of stable very unequal dual dynamic equilibrium arises with one or a few very rich farmers--"landlords"--and the rest poor farmers--"peasants". A large disturbance can "flip" the economy from stable equal distribution equilibrium to the stable dual distribution equilibrium.

If land per capita exceeds the critical value, equal distribution becomes an unstable dynamic equilibrium position. A stable dynamic equilibrium exists only at an unequal dual distribution.

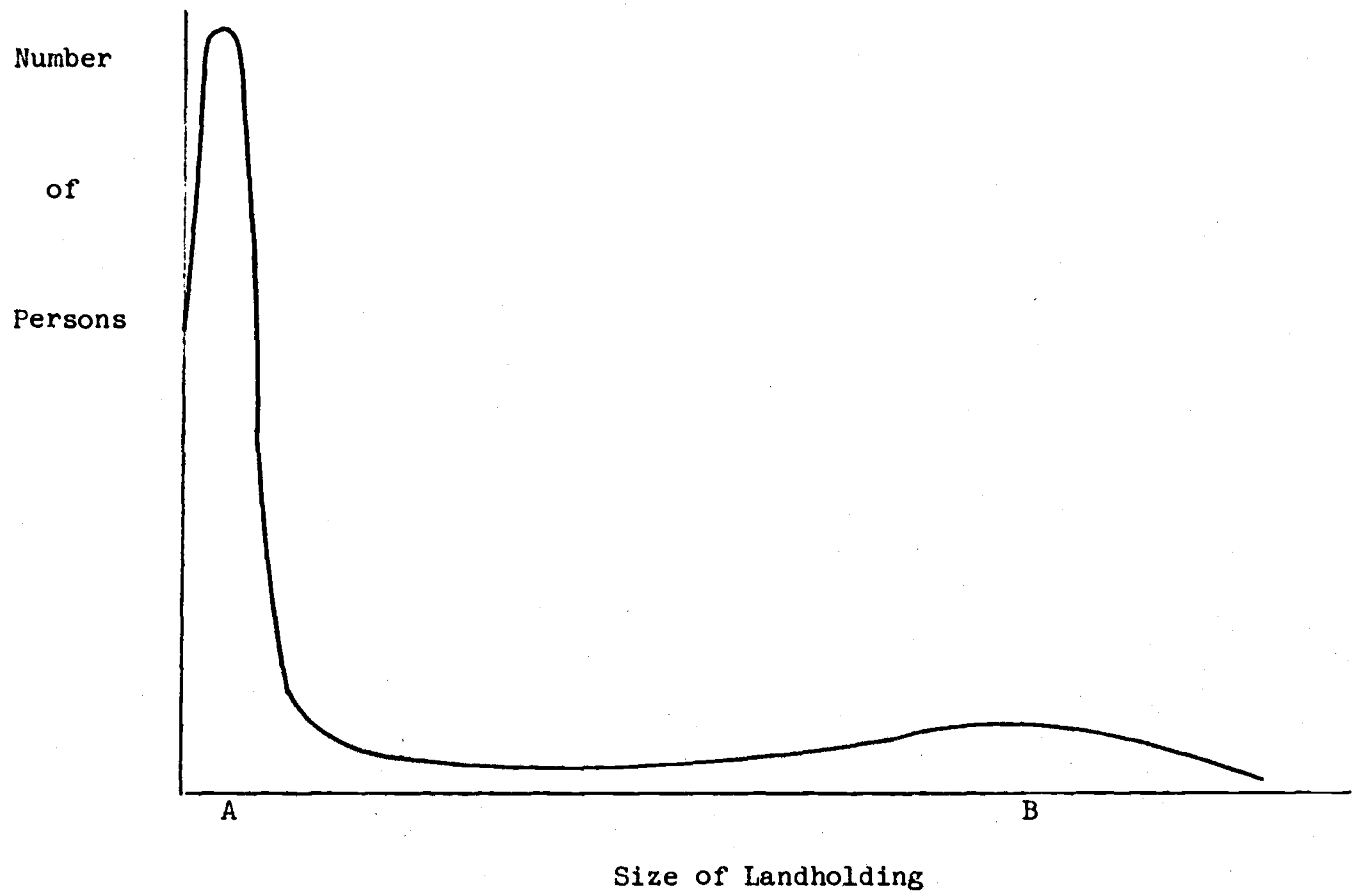
A dual distribution means there are two positions of dynamic equilibrium, that is, two separate levels of wealth where individuals keep the same wealth from period to period at the market land price. The lower position is individually stable; persons displaced from it save or dissave their way back to it. The upper position is individually unstable; persons displaced from it save or dissave further and further from it. But if there is only one person, or a few persons acting in concert at the upper position, they can affect the price of land enough to make the upper position stable when combined with the lower position.

Allowing for some random disturbances, dual distribution in the Clone economy should look rather like Fig. 8.1. The peasants cluster tightly around the lower postion, A. Because their position is individually unstable, and presumably they have difficulty collaborating, the landlords smear themselves widely around the upper position, B. So distribution in the Clone economy resembles real-world distribution both in the tendency to dualism and the very long upward "tail".

Causes and Consequences of Greater Inequality:

As noted, an increase in land per capita and/or size of population may shift an equal dynamic equilibrium to an unequal one. Such an increase also makes an existing unequal distribution more unequal. An improvement in technology that increases the output per acre, or lessens the diseconomies of scale due to transactions costs, also makes the dynamic equilibrium distribution more unequal. Finally, the more future-orientation increases with wealth, the more unequal the dynamic equilibrium distribution.

As distribution becomes unequal, or more unequal, the price of land rises. Table 8.1 shows what happens to selected economic variables as distribution moves from unstable dynamic equilibrium at equal distribution to stable dynamic equilibrium at unequal distribution. Notably, although the average discount rate rises, due to the numerical predominance of peasants, the weighted or "social" discount rate falls, --due to the overall predominance of the landlords.



<u>Fig. 8.1</u>: Number of persons plotted against size of landholding. A is a position of intrinsically stable dynamic equilibrium. B is a position of intrinsically unstable dynamic equilibrium, which becomes stable only when combined with A, and with a relatively small number of persons at B. If individuals are from time to time displaced a small distance from A or B, they return faster to A, making a sharper peak at A than at B.

Table 8.1

Changes as Distribution Moves from Equal to Unequal Dynamic Equilibrium

1. Price of land:	+
2. Discount rate and return on investment:	
Peasants:	+
Landlords:	-
Peasants' - landlords':	+
Average:	+ a)
Weighted average or "social rate of discount":	-
3. Gross output = income = profit = consumption:	
Peasants:	-
Landlords:	+
Total:	-
4. Potential income = liquidation value of firm:	
Peasants:	-
Landlords:	+
Total:	- ?
5. Wealth = present value of firm:	
Peasants:	-
Landlords:	+
Total:	+
6. Income/wealth = capital turnover:	
Peasants:	+
Landlords:	-
Total:	-

a) Assuming peasants dominate a simple average.

The models in this chapter permit only exogenous "growth", due for example, to changes in the production function. There is no net investment. But clearly, were the models altered to permit endogenous growth, the more unequal the distribution, the lower the potential rate of growth from a given net investment.

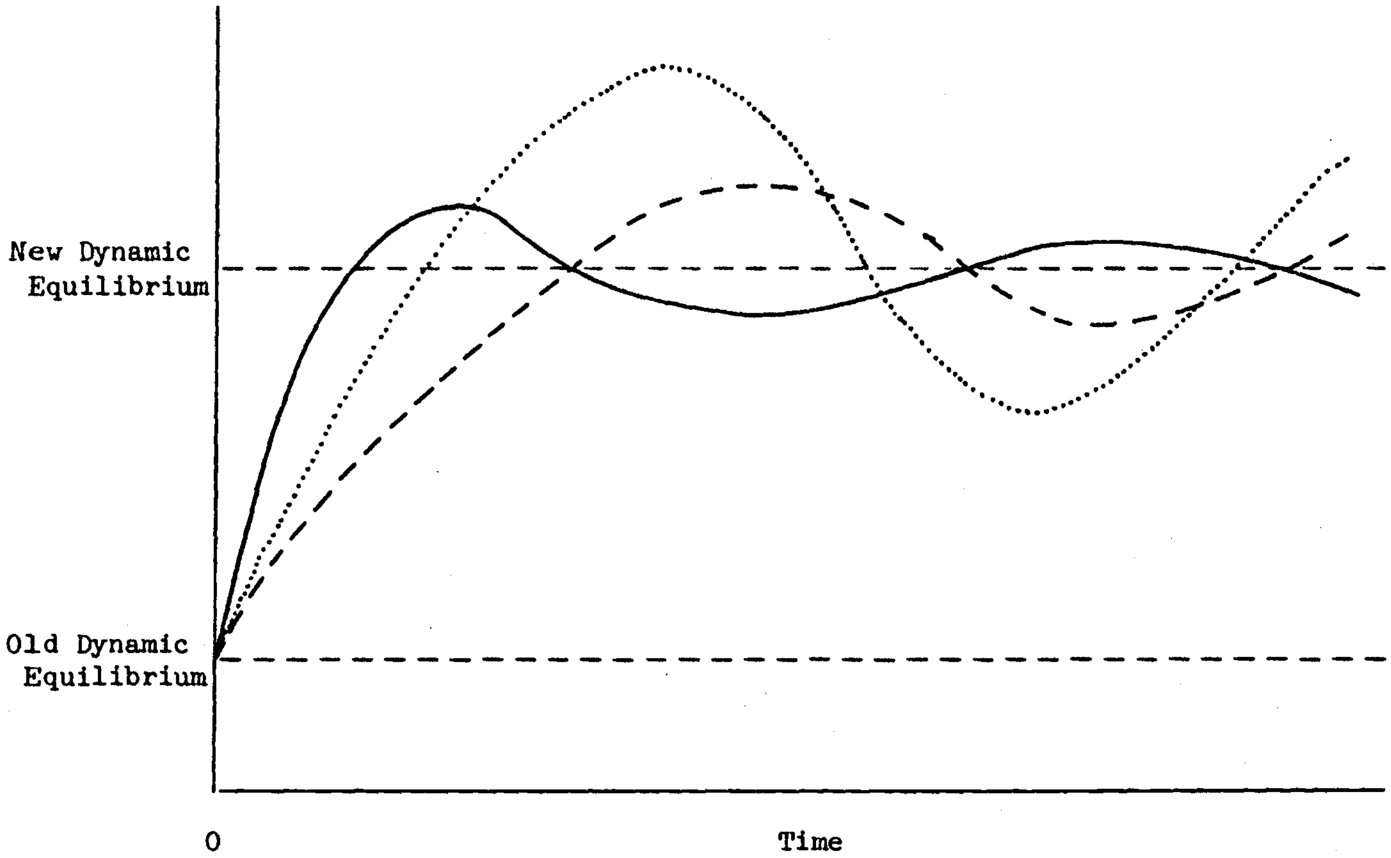
Growth and the Path to Dynamic Equilibrium:

Growth, whether exogenous or endogenous, shifts the position of dynamic equilibrium from its current location to a position of greater inequality. It sets the landlords to buying land from the peasants, moving both towards the new position.

But a system, like a weight on a spring, may oscillate about a position of dynamic equilibrium. Similarly, growth may start the landlord-peasant system to oscillating about the new equilibrium position. It suffices that there be a lag in the peasants' and landlords' perception of growth.

Fig. 8.2 shows a typical run of a computer model of a two-person economy, one peasant and one landlord. "Growth"--an increase in per acre productivity over a preset number of periods--shifts the dynamic equilibrium distribution to a new more unequal position. However, the system oscillates a while around the new position, with total output, land price, and distribution out of phase with each other.

Different assumptions about parameters produce different results. For some assumptions, damping prevents any overshooting of the new equilibrium position. Other assumptions produce explosive oscillations, --making the computer program "crash".



Total output of economy -- solid line
Size of landlord's land -- dashed line
Land price -- dotted line

Fig. 8.2: Consequences of growth. Improved technology increases output (solid line), increases landlord's land size (dashed line), and raises price of land (dotted line). Lag in perception of growth causes oscillations about the new dynamic equilibrium.

Future-Orientation and Redistribution:

Chps. 4 and 8 assume future-orientation rises monotonically with wealth. As noted in Chp. 4, it would be more realistic to assume that average future-orientation rises with wealth. So at any given wealth some people are more future-oriented than average, and are therefore saving, while others are less future-oriented than average and are therefore dissaving.

Chps. 4 and 8 also assume everyone has the same set of wealth-dependent preferences, that is, the identical utility map. It would be more realistic to assume future-orientation depends on past wealth as well as current wealth. That is, time preferences are learned, and change more slowly than external circumstances. Then people have genuinely different sets of preferences, depending on their histories.

One can even hypothesize a simple feedback relationship between wealth and time-preferences: Richer people have greater control over future consumption than do poorer people. Therefore, they learn to care more about the future than do poorer people. (There's no sense caring about what one can't control--hence the fatalism of the poor. Much more on this in Chp. 9). Moreover, people who happen to be more future-oriented than average get richer, and vice versa.

Finally the psychological literature shows that people unconsciously pick up most of their views from regular associates, and of course prefer to associate with those who share their views. The longer and more closely a group of people associates--in a family, a neighborhood, a club, or at work--the more their views converge. In effect, they come to share a "culture". [Blake, Mouton, 1981].

Recall that transactions costs give people of similar wealth good practical reasons to associate preferrentially with one another. So suppose upper levels of wealth do contain disproportionate numbers of people who learned greater future-orientation from direct experience, or got rich because they just happened to be more future-oriented. Then their attitudes rub off on family members and other associates, who come primarily from the same background. Vice versa for lower wealth levels. The result: distinct differences in class "culture", including time-preferences.

So here are two powerful forces for inequality. First, a rise in future-orientation with wealth can make inequality a position of stable dynamic equilibrium. Second, differences in wealth can reinforce differences in time-preferences, building them into class culture.

Consequently inequality may rapidly reappear following even the most radical revolution. Mao Zedong's Cultural Revolution appears to have been an attack on such reemerging inequality.

On the other hand, persistent redistributive efforts--public health, public education, income supports--may reduce class differences in time-preference, by reducing actual inequality. Such redistributive efforts then become self-reinforcing, for the less the class differences in time-preferences, the more equal the position of dynamic equilibrium. Public education may have a particularly great impact, as it not only redistributes wealth in the form of human capital, but makes a society's culture more uniform.

The developed countries have for generations pursued redistributive policies to varying degrees; most less developed countries have not. This difference perhaps helps explain why alot of growth in the developed countries has not apparently increased inequality; while only a little growth in less developed countries in recent years appears to have greatly increased inequality. (Of course, socialist governments have a propensity to redistribute so clumsily as to virtually kill all incentives to work or invest--but that's another story).

8.2 Summary of Sections in Chp. 8[B]

Sec. 8.3 describes the conditions necessary for individual and general dynamic equilibrium in a simple economy, and shows when such equilibrium is stable or unstable. In this economy, the dynamic equilibrium land price is the price at which farmers neither buy nor sell land. If diminishing returns to scale dominate at small wealth, and increasing future-orientation dominates at large wealth, then the dynamic equilibrium price as a function of land size falls and then rises again in a "U". The "critical land size" corresponds to the lowest price at the bottom of the "U". This "U" means farmers of different land size can be in dynamic equilibrium at the same market price of land.

Sec. 8.4 works out the conditions for equal and unequal dual dynamic equilibrium in a two person economy. To make a stable dual equilibrium possible, the "U" must be steeper on the left than on the right. Then, if the two farmers between them own less than or equal to twice the critical land size, only equal distribution is stable, or possible. If they own more land, equal distribution is unstable, while dual distribution becomes stable.

Sec. 8.5 presents a computer simulation of equal and unequal dual dynamic equilibrium in a two person economy, with results described above. In each period, the peasant and landlord buy or sell land, with supply and demand depending on each one's current consumption, and estimated future wealth. Estimated future wealth in turn depends on the projected rate of land price increase or decrease, derived from a running average or past price changes. "Growth" is simulated by an increase in per acre productivity or reduction in diseconomies of scale over a number of periods. Since growth raises the price of land, the projected rate of price change lags the actual rate of growth, causing oscillations for a long enough lag.

Sec. 8.6 shows the conditions for equal and unequal dual dynamic equilibrium in a multi-person economy, with results described above.

8.3 Individual and General Dynamic Equilibrium[C]

When is the self-sufficient, multi-period farmer of Sec. 4.4 in individual dynamic equilibrium at the same wealth? When can a whole economy of such farmers be in general dynamic equilibrium? When are such equilibria stable?

For Sec. 4.4 farmers, there are three kinds of equilibrium:

1. Static equilibrium. Each period, each farmer and the whole economy is in static equilibrium. That is, at the market price for land, supply equals demand for each farmer and the whole economy.

2. Individual dynamic equilibrium. Suppose a farmer, acting as a price-taker, keeps the same land from period to period. Then he is in individual dynamic equilibrium at that land size and land price.

3. General dynamic equilibrium. If all the farmers in an economy, whether of the same or varying wealth, keep constant land size from period to period at the market price determined by their total supply and demand--that puts the economy in general dynamic equilibrium.
If the economy is in general dynamic equilibrium, then each of the farmers must be in individual dynamic equilibrium, and vice versa.

A position of dynamic equilibrium, individual or general, may be stable, unstable, or neutral. (By analogy, a ball is in stable, unstable or neutral equilibrium, depending on whether it rests in a pit, atop a hump, or on a flat table).

The Dynamic Equilibrium Land Price:

Sec. 4.4 assumed a constant price of land, c, throughout an economy of self-sufficient farmers of varying wealth. Sec. 4.4 showed that if distribution of wealth remained the same from period to period, because no farmers wanted to buy or sell land at price c, then richer farmers had to be more future-oriented in their time preferences than poorer farmers. (Sec. 4.3 defines future-orientation).

If no farmers, whatever their wealth, want to buy or sell land at price c--that means that future-orientation not only increases with wealth, but increases at precisely the right rate to keep all farmers happy at c. This is rather an extreme assumption. It is much more plausible that future-orientation increases in such a way that c is the right price for farmers of only one or two different sizes of land. Other size farmers want to buy or sell. If so, dualistic distributions of wealth can become positions of dynamic equilibrium.

To show the relationship between price of land, land size, and time-orientation, consider again a Sec. 4.4 farmer who behaves according to equation (4.4.15):

$$(4.4.15) \quad m(g(T_o)+c(T_o-T_1),W_1) - 1 - \frac{g'(T_1)}{c} = 0$$

Assume that $\left.\frac{dW_1}{dc}\right|_{T_1} > 0$. That is, next period wealth, W_1, increases with an increase in land price, holding next period land constant.

Then if land price increases, holding constant the farmer's initial land size, T_o, he sells more or buys less land at the end of the period than he otherwise would. For:

$$(3.1) \qquad \left.\frac{dT_1}{dc}\right|_{T_o} = \frac{-\left[m_o(T_o-T_1) + \frac{g'(T_1)}{c^2} + m_1 \frac{dW_1}{dc} \right]}{-m_oc + m_1[c + g'(T_1)] - \frac{g''(T_1)}{c}} < 0$$

The amount of land demanded in the next period, T_1, falls as the price of land rises.

(The expression is obviously < 0 for $T_1 \geq T_o$, and for some region

$T_1 < T_o$ since the 2nd two terms in the denominator are > 0. Conceivably though improbably, for some $T_1 << T_o$ at very high price, an income effect makes demand for land rise again as price continues to rise).

It follows from equation (3.1) that for each present quantity of land, T_o, there is a particular price, $e(T_o)$ at which the farmer chooses to keep just T_o acres in the next period. So since at price $e(T_o)$, $T_1, T_2, \ldots = T_o$, $e(T_o)$ can be written simply $e(T)$ for all T. $e(T)$ is a function giving the price at which a farmer with land T keeps that size farm indefinitely. It is the price which keeps the farmer with land T in individual dynamic equilibrium. At land price $c > e(T)$, he sells land, while at land price $c < e(T)$, he buys land. (Sec. 4.4 simply assumed $e(T)$ to be constant for all T).

Fig. 8.3 shows a farmer in individual dynamic equilibrium at land price e*.

Conditions for Individual Dynamic Equilibrium:

What conditions must the land price $e(T)$ meet to keep a farmer in individual dynamic equilibrium? How does this land price change as the wealth of a farmer increases?

The equations for a farmer in dynamic equilibrium become (from (4.4.18) - (4.4.20)):

(3.2) $$y = g(T)$$

(3.3) $$W = (1 + \frac{1}{r})y$$

(3.4) $$m(y,W) = 1 + r = 1 + \frac{g'(T)}{e}$$

And from (4.4.22):

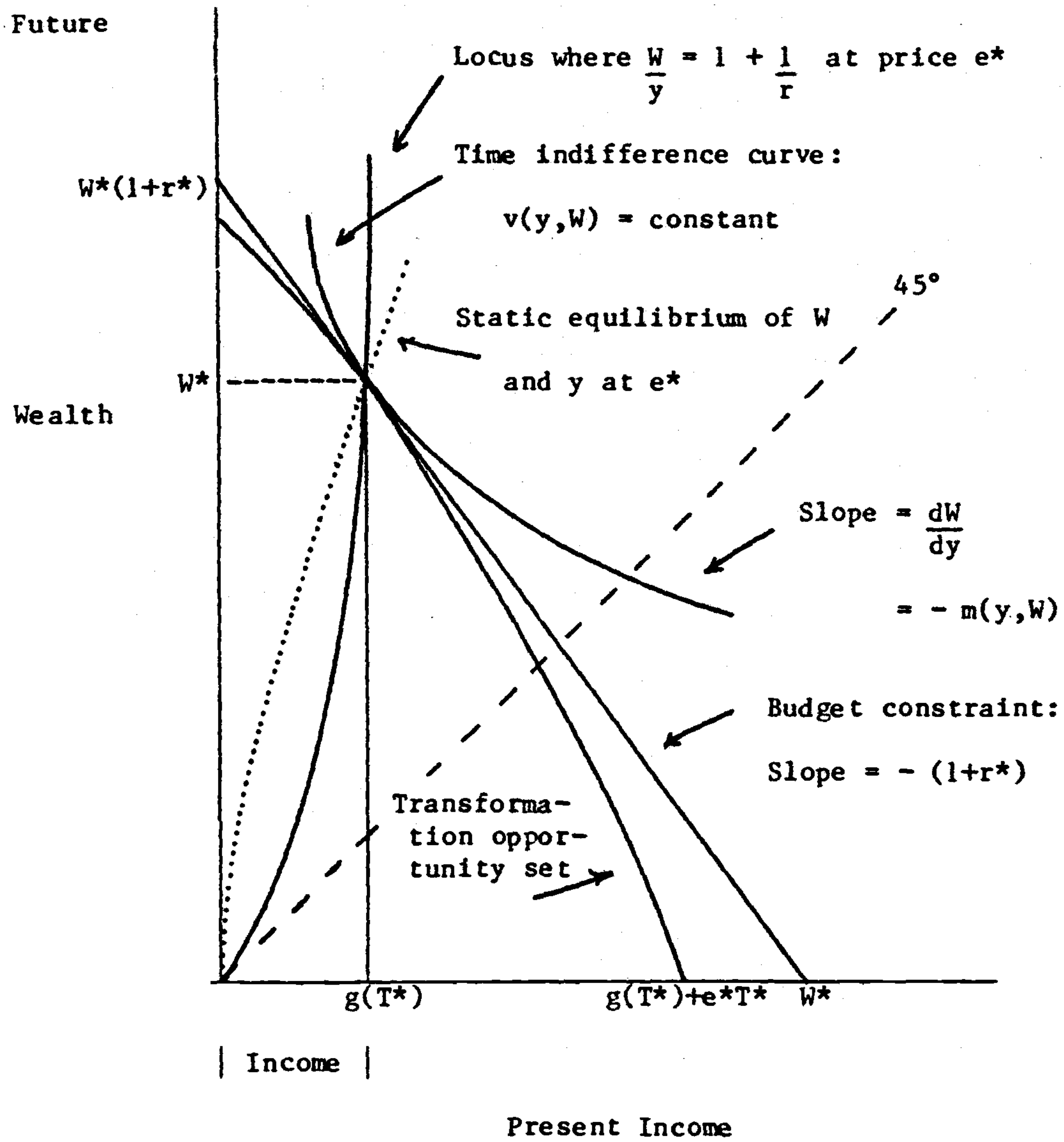


Fig. 8.3: The farmer maximizes utility at the point his time indifference curve lies tangent to his budget constraint and his transformation opportunity set, at land size T*.

Assume the farmer is on the no-sale locus for land price e*. That automatically puts him on the locus for which W/y = 1 + 1/r, at land price, e*, so land size, T*, wealth, W*, income, y* = g(T*), and discount rate r* = g'(T*)/e* remain constant.

The dotted line shows the locus of static equilibrium combinations of y and W at price e*, that is, the locus of points of tangency between indifference curves and the transformation opportunity set at land price e*. To the left of y* and W*, the farmer buys land, while to the right he sells land, making y* and W* a point of stable dynamic equilibrium.

The solid line marking the locus where W/y = 1 + 1/r does not correspond to points of static equilibrium at land price e* except at the no-sale point, y* and W*.

(3.5) $$\left.\frac{dm}{dy}\right|_t = m_o + (1+\frac{1}{r})\, m_1 \quad \begin{matrix} = 0 & \text{constant time preferences} \\ < 0 & \text{increasing future-orientation} \end{matrix}$$

Substituting from (3.2) and (3.3), (3.4) becomes:

(3.6) $$m[g(T), g(T)(1+\frac{e}{g'(T)})] - 1 - \frac{g'(T)}{e} = 0$$

(3.6) can be solved implicitly for the dynamic equilibrium price, e(T). Then:

(3.7) $$\frac{de}{dT} = \frac{1}{1 + \frac{m_1 g}{r^2}} \left[\underbrace{- e^2(m_o + (1+\frac{1}{r})m_1)}_{1} + \underbrace{\frac{g}{r}''(1 + \frac{m_1 g}{r^2})}_{2} \right]$$

From (3.5), above, the first term = 0 if time preferences do not change with wealth, and > 0 (due to the - sign) if future-orientation increases with wealth.

With transactions costs, so $g'' < 0$, the second term is < 0. Without transactions costs, $g'' = 0$, so the second term = 0.

This allows four possibilities:

1. Time preferences don't change with wealth, but transactions costs cause decreasing returns in production. Then the dynamic equilibrium land price e(T) simply falls as T increases.

2. Future orientation increases with wealth, but, absent transactions costs, production shows constant returns. Then the dynamic equilibrium land price e(T) simply rises with wealth.

3. Time preferences remain constant with wealth, and production shows constant returns. Then land price remains constant.

4. Future orientation increases with wealth, and there are diminishing returns in production. Now the direction of price

change becomes ambiguous.

In this case, there are a priori any number of possible patterns of price as a function of land size--including constant price as assumed in Sec. 4.4. But since only two factors determine the price pattern, scale in production technology and future-orientation, it's reasonable to assume only a simple pattern results:

a. A steady decline, if decreasing returns dominate, as in 1. above.

b. A steady rise, if increasing future-orientation dominates, as in 2. above.

c. A "U", if decreasing returns dominate at small scale, (due to much greater productivity of small holdings), while increasing future-orientation dominates at larger scale.

d. An inverted "U", for the opposite of c.

Obviously, only a "U" or inverted "U" permit farmers of different land size to be in individual dynamic equilibrium at the same price.

Stability of Individual Dynamic Equilibrium:

Suppose a farmer owns land T^* at price $e(T^*) = p$, a given market land price. Then he is in stable, unstable or neutral dynamic equilibrium according to whether $e(T)$ is declining, rising, or constant at p, ie., whether $e'(T^*) < 0$, > 0, or $= 0$.

Stable equilibrium for $e'(T^*) < 0$. Suppose we give the farmer a bit more land, dT. Then the actual market price $p = e(T^*) > e(T^*+dT)$, the price at which he would keep just T^*+dT. So over the next few periods, he sells land, returning to his initial landholding, T^*. Conversely, if we take a bit of land from the farmer, he buys land, returning to T^*.

Unstable equilibrium for $e'(T^*) > 0$. If the farmer owns T^* at market price p, and we give him dT more, then $p = e(T^*) < e(T^*+dT)$. So he continues to buy more land, moving farther and farther from T^*. He goes on and on selling land if we take dT from him.

Neutral equilibrium for $e'(T^*) = 0$. If he owns T^* at $e(T^*)$, and we give him dT more, he just stays at T^*+dT.

General Dynamic Equilbrium:

When can two or more farmers with the same or different size land be in individual dynamic equilibrium at the same land price--making a general dynamic equilibrium? When is such a general dynamic equilibrium stable, unstable, or neutral?

Equal Distribution.

Equal distribution is always a position of general dynamic equilibrium. Only equal distribution is possible if e(T) falls everywhere, as in 1. or 4a.; or rises everywhere, as in 2. and 4b. above.

Suppose there are E acres of land in the economy, and N farmers. Each farmer owns E/N acres, in individual dynamic equilibrium at a market land price, e(E/N).

If $e'(E/N) < 0$, equal distribution clearly is a stable position of general dynamic equilibrium. That is, if we transfer a bit of land dT from one farmer to another, they will buy and sell their way back to E/N acres apiece. If $e'(E/N) = 0$, equal distribution is neutral general dynamic equilibrium; and if $e'(E/N) > 0$, it is unstable.

From this it follows that:

If $e'(T) < 0$ everywhere, as in case 1., or 4a. above, then since equal distribution is stable, any initial distribution moves over time to equality. That is, in a world where transactions costs create diminishing returns to scale, and future-orientation does not increase with wealth, or diminishing returns everywhere outweigh increasing future-orientation--then distribution must continually move toward equality.

If e'(T) > 0 everywhere, as in case 2. and 4b. above, then any initial distribution must become more and more unequal over time until one or more farmers own everything, and the rest own nothing. That is, if increasing future-orientation with wealth everywhere outweighs diminishing returns to scale--then distribution must continually become less equal.

If e'(T) = 0 everywhere, as in case 3, then any initial distribution just stays that way. Without transactions costs and with constant time-preferences, distribution reflects, like craters on the moon, all historical distributing events. If these are random, then distribution forms a bell-shaped curve.

Dual Distribution.

Dual distribution requires that price e(T) fall and then rise, a "U", or rise and then fall, an inverted "U".

However, as will appear, only one pattern yields plausible predictions: a "U", with a steep left side and a gentle right size. This pattern appears in Fig. 8.4.

Sec. 8.4, next, examines the conditions for a stable two-person dual distribution. Sec. 8.5 presents a simple computer simulation of the two-person dual distribution.

Sec. 8.6 examines the conditions for a stable multi-person dual distribution.

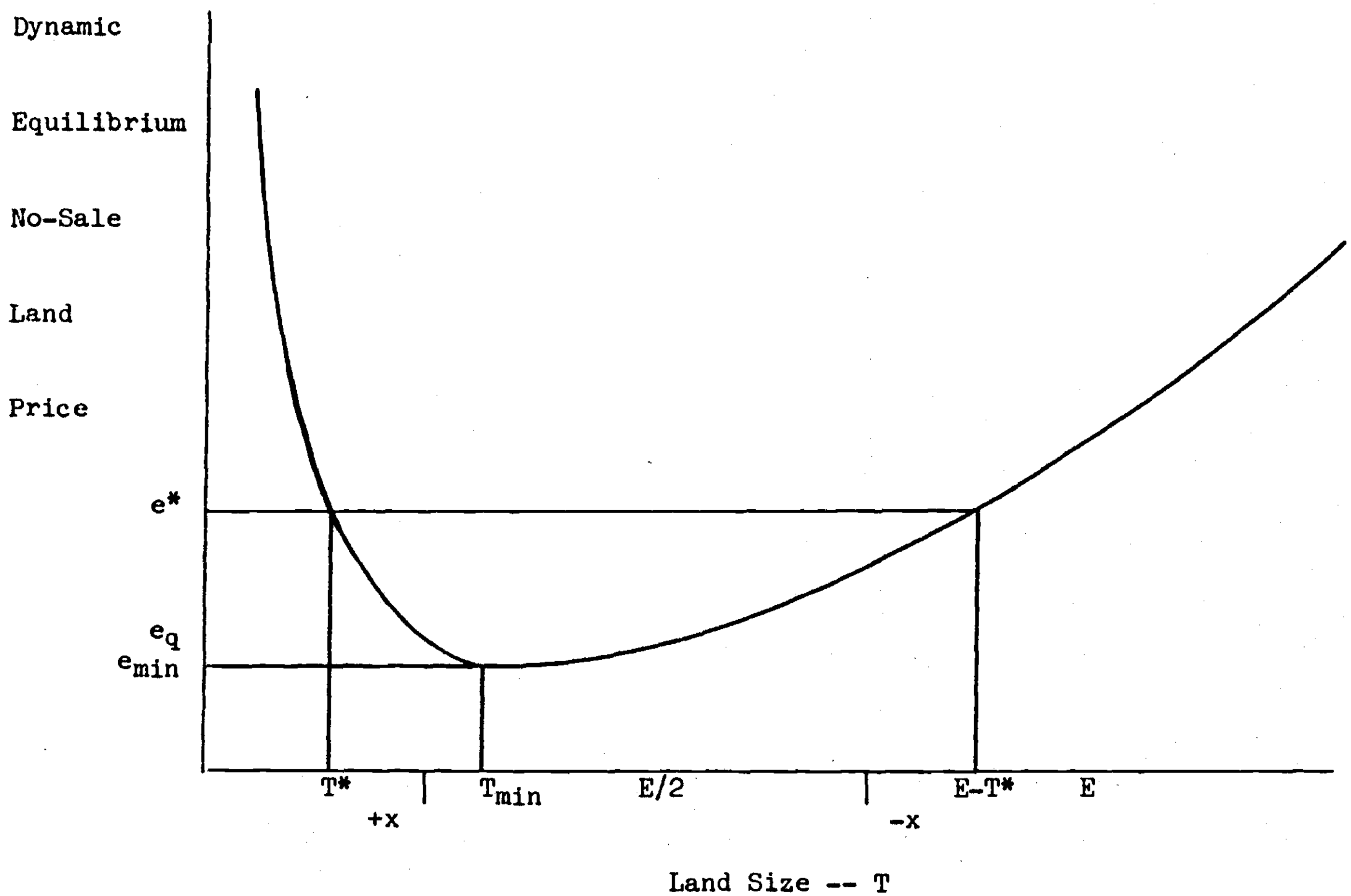


Fig. 8.4: Dynamic equilibrium no-sale price as a function of land size. Given two landholders, and total land in the economy, E, a division of land T* and E-T* is a stable dynamic equilibrium. Take an area of land, x, from the landlord at E-T*, and give it to the peasant at T*. Then the peasant sells land to the landlord until both return to their original position at T* and E-T*.

8.4 Landlord and Peasant: Two-Person General Dynamic Equilibrium

As in Chp. 7, imagine an economy with only two people: a landlord and a peasant. Both behave as self-sufficient farmers according to equation (4.4.15). They can buy or sell land between themselves. And while neither acts as monopolist or monopsonist, their supply and demand for land determine the market price.

Then landlord and peasant can achieve a stable unequal dynamic equilibrium distribution if and only if:

a. The dynamic equilibrium price as a function of land size, e(T), forms a "U" steeper on the left side than on the right, as shown in Fig. 8.4.

b. Landlord and peasant between them own more than twice T_{min}, the quantity of land at the lowest equilibrium price, e_{min}, at the bottom of the "U".

If landlord and peasant own less than $2T_{min}$, only equal distribution is stable, or even possible. If they own more than $2T_{min}$, equal distribution is also a possible equilibrium, but unstable. The more land in the economy, the more unequal the stable dynamic equilibrium distribution.

As the economy moves from an unstable dynamic equilibrium at equal distribution, to a stable one at unequal distribution, the price of land rises.

The Landlord and Peasant in General Dynamic Equilibrium:

Referring to the "U" curve in Fig. 8.4, suppose the landlord and peasant between them own an area of land E. If we distribute land equally between them, each has E/2, and the 'price of land in the economy must be e(E/2).

As shown in Sec. 8.3, E/2 is a position of stable individual and general dynamic equilibrium if e(E/2) lies on the downward-sloping part of the "U", that is, if $E/2 < T_{min}$ at the bottom of the "U". $E/2 = T_{min}$ is also stable, due to the greater steepness of the left side of the "U". Also due to greater steepness on the left, when $E/2 \leq T_{min}$ unequal distribution is not even possible. That is, for $E/2 \leq T_{min}$, there is no way to divide E unequally between landlord and peasant to put them in dynamic equilibrium at the same price for land.

E/2 is a position of unstable individual and general dynamic equilibrium if e(E/2) lies on the upward-sloping part of the "U", that is, if $E/2 > T_{min}$. But in this case, there exists a position of unequal dynamic equilibrium as shown in Fig. 8.4: The peasant owns $T^* < T_{min}$ acres and the landlord owns $E-T^* > T_{min}$, such that $e(T^*) = e(E-T^*) \equiv e^*$. That is, e^* and T^* must simultaneously solve the peasant's and landlord's versions of (3.6):

$$(4.1) \qquad m[g(T), g(T)(1+\frac{e}{g'(T)})] - 1 - \frac{g'(T)}{e} = 0$$

$$m[g(E-T), g(E-T)(1+\frac{e}{g'(E-T)})] - 1 - \frac{g'(E-T)}{e} = 0$$

The price, e^*, at unequal distribution, exceeds the price e_q at equal distribution. And $e_q > e_{min}$, the price at the bottom of the "U".

So T^* and $E-T^*$ is a position of unequal dynamic equilibrium. From Sec. 8.3, T^* is individually stable, holding land price constant, while $E-T^*$ is individually unstable. Whether or not the combination is stable or unstable depends on how a displacement from T^* and $E-T^*$ affects the price of land.

In fact, the combination is stable only given the assumption that the left side of the "U" is steeper than the right, as the following argument shows:

Recall from Sec. 8.3 that if any landowner owning T acres finds himself facing a market land price higher than e(T), he sells land. Facing a price lower than e(T), he buys land.

Now start at the unequal distribution equilibrium where the peasant owns $T^* < E/2$, while the landlord owns $E-T^*$. Take a bit of land, x, from the landlord and give it to the peasant, leaving landlord and peasant with $E-T^*-x$ and T^*+x, respectively. Because the "U" is steeper at T^* than at $E-T^*$, $e(E-T^*-x) > e(T^*+x)$.

But $e^* > e(E-T^*-x) > e(T^*+x)$, so at first both want to sell land, driving down the price. Eventually, the price reaches the landlord's dynamic equilibrium price $e(E-T^*-x)$, at which the landlord no longer wants to sell land, and below which he wants to buy. But the peasant goes on bidding down the price until they reach static equilibrium somewhere between $e(E-T^*-x)$ and $e(T^*+x)$, and the peasant sells a bit of land to the landlord.

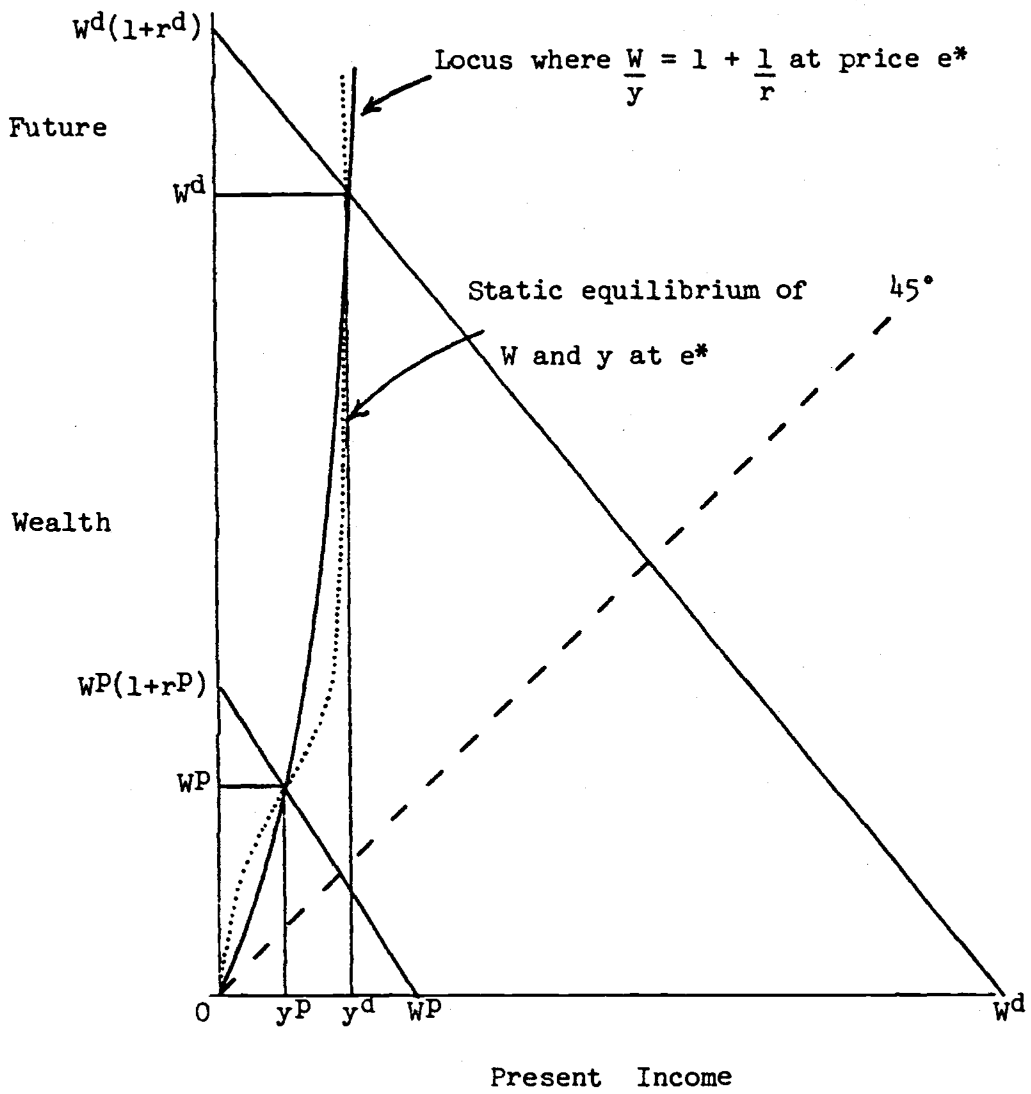
In following periods, they repeat the same process, gradually buying and selling their way back to the dynamic equilibrium position at T^* and $E-T^*$.

That makes T^* and $E-T^*$ a position of <u>stable</u> general dynamic equilibrium.

Fig. 8.5 shows the dynamic equilibrium between peasant and landlord as a function of the peasant's income, y^p, and the landlord's income, y^d.

Other Possible Dynamic Equilibrium Price Curves:

An unequal dynamic equilibrium requires a "U" shaped or inverted



<u>Fig. 8.5</u>: Two-person dynamic equilibrium at price e*. Notice that the static equilibrium locus at e* (dotted line) <u>twice</u> crosses the locus of $W/y = 1 + 1/r$ for e* (solid line), from left to right at a steep angle, and back again at a shallow angle.

On the e* static equilibrium locus to the left of the locus of $W/y = 1 + 1/r$, a person wants to buy land, while on the right a person wants to sell. So the peasant's position at y^p and W^p is stable, while the landlord's position at y^d and W^d is unstable. However, the combination forms a stable dynamic equilibrium, as suggested by the difference in angle of crossing.

$y^p = g(T^*)$ and $y^d = g(T^*-E)$. Notice that while $y^d = 2y^p$, $W^d = 4W^p$. The slopes as drawn show $r^p = .5$, while $r^d = .2$. So $W^p/y^p = 1 + 1/.5 = 3$, while $W^d/y^d = 1 + 1/.2 = 6$.

"U" shaped "no-sale" price curve. So that leaves three possibilities besides the one just discussed: a. a "U" falling gently and rising b. an inverted "U" rising steeply then falling gently, and c. an inverted "U" rising gently then falling steeply. All three yield bizarre preditions.

Possibilities a. and b. produce an unstable dual dynamic equilibrium, together with a stable dynamic equilibrium at equality and total inequality.

Possibility c. produces a stable unequal dynamic equilibrium and an unstable equal dynamic equilibrium. However, (as with possibility a.), unequal distribution is possible only for total land in the economy less than twice T_{min}. The less land in the economy, the more unequal the distribution.

An inverted "U" price curve also may produce negative prices.

Effects of Shift from Equal to Unequal Dynamic Equilibrium:

Table 8.2 summarizes the principal effects of a shift from equal to unequal dynamic equilibrium. Some comments on Table 8.2:

2. Discount rate and return on investment. The peasant's discount rate rises even though land price, e, rises, because:

$$(4.2) \qquad \frac{dr}{dT} = \frac{dm}{dT} = \frac{er}{1 + \frac{m_1 g}{r^2}} \left[m_0 + \left(1 + \frac{1}{r}\right) m_1\right] \begin{array}{ll} < 0 & \text{greater fut-orient} \\ = 0 & \text{const. time pref.} \end{array}$$

It is assumed that the peasant dominates a simple average--as peasants would clearly dominate in an economy with many peasants and few landlords. The first weighted average, r*, comes out the same if discount rates are weighted by land size, T^p and T^d, and divided by total land, E; or if discount rates are weighted by investment eT^p and

Table 8.2

Changes as Distribution Moves from Equal to Unequal Dynamic Equilibrium

1. Price of land: e	+
2. Discount rate and ROI: $r = g'/e$	
Peasant: $r^p = g'(T^p)/e$	+
Landlord: $r^d = g'(T^d)/e$	-
Peasant - Landlord: $r^p - r^d$	+
Average: $(r^p + r^d)/2$	+ a)
Weighted average: $r^* = [g'(T^p)T^p + g'(T^d)T^d]/eE$	-
Alt. wgtd avg: $\frac{1}{r^\dagger} = \frac{g(T^p)/r^p + g(T^d)/r^d}{g(T^p) + g(T^d)}$	-
3. Gross output = income = profit = consumption:	
Peasant: $y^p = F^p = g(T^p)$	-
Landlord: $y^d = F^d = g(T^d)$	+
Total: $y^p + y^d = F^p + F^d = g(T^p) + g(T^d)$	-
4. Potential income = liquidation value:	
Peasant: $g(T^p) + eT^p$	-
Landlord: $g(T^d) + eT^d$	+
Total: $g(T^p) + g(T^d) + eE$	- ?
5. Wealth = present value of firm:	
Peasant: $W^p = (1+1/r^p)y^p$	-
Landlord: $W^d = (1+1/r^d)y^d$	+
Total: $W = W^p + W^d = (1+\frac{1}{r^\dagger})(y^p+y^d)$	+
6. Income/wealth = capital turnover:	
Peasant: $1 + 1/r^p$	+
Landlord: $1 + 1/r^d$	-
Total: $1 + 1/r^\dagger$	-

a) Assuming peasant dominates a simple average.

eT^d, and divided by total investment, eE. The second weighted average, $r^\dagger$, is such that the sum of the peasant's and landlord's wealth = (1 + $1/r^\dagger$) the sum of their incomes. Both weighted averages, r^* and $r^\dagger$, converge to the landlord's discount rate, r^d, as distribution becomes more unequal. The weighted averages could be said to measure some sort of "social discount rate".

3. Gross output = income = profit = consumption. The omission of labor and labor income in the simplified model of Sec. 4.4 makes gross output equal income and profit. It also equals consumption in each period. Total gross output falls due to diseconomies of scale, as shown in Chp. 1, Sec. 1.5.

4. Potential consumption = liquidation value. It equals gross output or ordinary income plus the value of land--hence, the liquidation value of the firm(s). (Of course, were the landowners in a two-person model to try to liquidate, they would drive down the price).

5. Wealth = present value of firms. Notice that wealth exceeds the liquidation value of firms.

6. Income/wealth. The ratio of income to wealth equals simply 1 + 1/r, in dynamic equilibrium.

7. Capital turnover. The peasant's turnover presumably increases due to the increase in output per acre, despite the rise in land price.

Economic Growth and Stability:

The models presented so far assume no growth. All investment goes for replacement. There is no net investment. There is also no exogenous technological change.

Consider now what happens if technology improves, either due to net investment, or exogenous factors. Say an improvement in technology raises output per acre, or lessens diseconomies of scale. If the pattern of time preferences does not change, then this improvement clearly shifts a stable unequal dynamic equilibrium distribution toward greater inequality. It may also make a stable equal distribution equilibrium unstable, so that the economy eventually moves to unequal distribution.

A stable dynamic equilibrium exists at a point if the system returns to it after a small disturbance. However, when the disturbed system returns to this point, it may exhibit another kind of instability: oscillations--just as a weight on a spring may bounce up and down when disturbed. Consequently economic growth, by shifting the general dynamic equilibrium position, may cause economic oscillations.

8.5 A Computer Simulation of Dynamic Equilibrium[C]

The two person dynamic equilibrium model described in Sec. 8.4 easily lends itself to simulation on a small computer. Such simulation requires explicit functional forms for the marginal rate of substition function and the production function. It also requires certain crude approximations to keep computations within the computer's capacity.

The Marginal Rate of Substitution Function:

The marginal rate of substition function m(y,W) can be modelled simply:

$$(5.1) \qquad m(y,W) = W(A/y - B) > 0 \quad \text{for} \quad 1/y > B/A$$

where A and B are constant parameters. B/A gives the "index of future-orientation". If B = 0, time preferences do not change with wealth. The higher B/A, the greater the increase in future-orientation with wealth.

This marginal rate of substitution function has the right properties:

$$(5.2) \qquad m_0 = - \frac{WA}{y^2} < 0$$

$$(5.3) \qquad m_1 = A/y - B > 0$$

$$(5.4) \qquad m_0 + \frac{W}{y} m_1 = - \frac{WB}{y} < 0$$

So the slope of time indifference curves flattens along a ray from the origin, unless B = 0.

The Production Function:

The production function, $g(T)$, can be modelled as a simple exponential function:

(5.5) $$g(T) = CT^D \qquad 0 < D \leq 1$$

where C and D are constant parameters.

This function also has all the right properties:

(5.6) $$g'(T) = CDT^{D-1} > 0$$

(5.7) $$g''(T) = -CD(1-D)T^{D-2} < 0$$

So there are diminishing returns in production unless $D = 1$.

Since $y = g(T) < A/B$, that makes the maximum size land anyone can own in this computer economy:

(5.8) $$T_{max} = \left(\frac{A}{BC}\right)^{1/D}$$

Dynamic Equilibrium Price as a Function of Land Size:

Equation (3.4) can now be solved explicitly for the dynamic equilibrium price, e, as a function of land size:

(5.9) $$e = \frac{CDT^{D-1}}{A - BCT^D} = \frac{CDT^D}{T(A-BCT^D)}$$

Price obviously approaches infinity as land size approaches either zero or T_{max}.

And the change in e as a function of land size T becomes:

(5.10) $$\frac{de}{dT} = \frac{CDT^{D-2}}{(A-BCT^D)^2}\left[BCT^D - (1-D)A \right]$$

This change is obviously < 0 for small T, and > 0 for large T.

The minimum no-sale price, e_{min}, occurs where de/dT = 0. Land at minimum price, T_{min}, is:

$$(5.11) \qquad T_{min} = \left(\frac{A(1-D)}{BC} \right)^{1/D}$$

And minimum price is:

$$(5.12) \qquad e_{min} = \left[\frac{C}{A} \left(\frac{1-D}{B} \right)^{1-D} \right]^{1/D}$$

Note that neither minimum land nor minimum price exist for B = 0.

The ratio of minimum land to maximum land is:

$$(5.13) \qquad \frac{T_{min}}{T_{max}} = (1-D)^{1/D} < \frac{1}{2}$$

This ratio depends only on D, the exponential coefficient of the production function. It is always less than 1/2, ranging from a maximum of 1/e (exp fn) for D near 0, to a minimum of 0 as D approaches 1.

So the "no-sale" price as a function of land size forms a "U" shaped curve, falling steeply to the minimum price and then rising gently. The arms of the "U" approach infinity for land size near 0 or near T_{max}.

A computer can easily plot this curve.

Dynamic Equilibrium Distribution in a Two-Person Economy as a Function of Total Land:

Given E, the total land in a two-person economy, it is possible to solve equations (4.1) for the dynamic equilibrium distributions as a function of the parameters A, B, C, and D.

A simple computer program gives the distribution as E ranges from just above zero to just below T_{max}, the maximum land anyone can own. The computer starts with an arbitrary unequal distribution of E between two persons. It finds the dynamic equilibrium distribution by solving by successive approximation an equation derived from (5.9):

$$(5.14) \qquad T^{1-D}[A - BCT^{D}] \; - \; (E-T)^{1-D}[A - BC(E-T)^{D}] \; = \; 0$$

As predicted, for $E \leq 2T_{min}$, the land at minimum price, the dynamic equilibrium lies at equal distribution. For $E > 2T_{min}$, the dynamic equilibrium distribution becomes unequal; the larger E, the greater the inequality. (An unstable equal distribution equilibrium of course exists for $E > 2T_{min}$. But when the computer starts at unequal distribution, it can only grope its way to a stable equilibrium).

Simulation of a Two-Person Land Market:

Using the MRS and production functions, the computer can simulate a two-person land market. Each period, one person buys land from the other, until they end up at the stable dynamic equilibrium distribution, equal or unequal.

Away from the dynamic equilibrium price e(T), the marginal rate of substitution function depends on present consumption C_o and next-period wealth, W_1, which change from period to period.

Present consumption can be modelled simply from (4.4.15):

$$(5.15) \qquad C_o \; = \; g(T_o) + c(T_o - T_1) \; = \; CT_o{}^{D} + c(T_o - T_1)$$

Next period wealth requires an approximation. At the dynamic equilibrium price, present and future wealth simply equals (1 + 1/r) times income (which equals consumption). But away from the dynamic equilibrium price, next period wealth depends on future consumption and discount rates from now until kingdom come.

Plausibly, next period wealth depends chiefly on next period production $g(T_1)$, and estimated rate of price change, x, discounted at the current discount rate, $r_o = g'(T_o)/c_o$, where c_o is the price in the last period. So W_1 is approximated:

$$(5.16) \qquad W_1 = [g(T_1) + xT_1](1+\frac{1}{r_o}) = [CT_1{}^D + xT_1](1+\frac{1}{r_o})$$

This approximation converges to W at dynamic equilibrium price, where $T_o = T_1 = T$, and $x = 0$. It makes sense that a positive estimated rate of land price change, x, raises future wealth, while a negative rate lowers future wealth.

The estimated rate of price change, x, in turn is calculated from a least squares fit to land prices in an arbitrary number of preceding periods.

Then the marginal rate of transformation of present period income into next period wealth becomes:

$$(5.17) \qquad -\frac{dW_1}{dy_o} = -\frac{dW_1}{dT_1}\frac{dT_1}{dy_o} = \frac{[g'(T_1) + x]}{c}(1 + \frac{1}{r_o})$$

$$= \frac{[CDT_1{}^{D-1} + x]}{c}(1 + \frac{1}{r_o})$$

In static equilibrium each period, this marginal rate of transformation must equal the marginal rate of substitution:

$$(5.18) \qquad m(y_o,W_1) = W_1(\frac{A}{y_o} - B)$$

$$= [g(T_1) + xT_1][\frac{A}{g(T_o)+c(T_o-T_1)} - B](1 + \frac{1}{r_o})$$

Static equilibrium in each period requires that marginal rate of transformation equal marginal rate of substitution for both landlord and peasant:

$$(5.19) \qquad -\frac{dW_1}{dy_o} - m(y_o, W_1) = 0$$

or, from (5.17) and (5.18)

$$(5.20) \qquad \frac{[CDT_1^{D-1} + x]}{c} - [CT_1^{D} + xT_1][\frac{A}{CT_o^{D} + c(T_o - T_1)} - B] = 0$$

The computer simultaneously solves (5.20) as written, with T_o and T_1 for the peasant's present and next period land, and (5.20) with $E - T_o$ and $E - T_1$ for the landlord's present and next period land. Note that the factor $(1+1/r_o)$ drops out. The computer solves these two equations for two variables: T_1, the peasant's land in the next period, and c, the land price that produces static equilibrium.

The landlord and peasant begin away from the dynamic equilibrium distribution for the particular parameters A, B, C, and D, and the total land between them, E. The computer calculates the static equilibrium distribution for landlord and peasant in each period, as well as the price of land and the quantity of land transferred from one to the other. The next period it starts with the new static equilibrium distribution and price, and repeats the calculation.

As it should, the static equilibrium distribution converges toward the dynamic equilibrium distribution. If the estimated rate of price change, x, depends on a large enough number of lagged prices, the static equilibrium path may overshoot and oscillate around the dynamic equilibrium position before converging. For some choices of parameters the oscillations may explode instead of damping out. When the land transfers and price changes become very small, the computer quits.

Economic Growth and Stability:

This simulation permits modelling economic growth in three different ways:

a. The total amount of land in the economy, E, may increase.

b. The per acre productivity of land may increase. That is, if $g(T) = CT^D$, then the parameter C increases.

c. Diminishing returns may lessen. That is, the parameter D increases.

All three forms of "growth" make the dynamic equilibrium distribution more unequal,--as apparent from derivatives of (5.14) with respect to C, D, and E. "Growth" also increases output and land price. However, output does not increase as much as it would have if distribution remained the same, while price increases more than it would have.

Moreover, the movement from one dynamic equilibrium position to another, due to growth, may produce oscillations in output, distribution, and land price. Such oscillations occur, given a sufficient lag in the "perception" of growth.

Since "growth" raises the price of land, the computer models the "perception" of growth in the estimated rate of price change, x. This estimated rate of price change is just the slope of a least squares fit to a selected number of lagged prices. In other words, the landlord and peasant estimate the rate of price change by projecting recent trends. The greater the number of lagged prices, the more the estimated rate of price change reflects earlier price changes.

So the computer model of "growth" proceeds as follows: The landlord and peasant start at a position of unequal dynamic equilibrium. One of the parameters, C or D, changes in increments over a selected number of periods. The landlord and peasant immediately set off to seek the new more unequal dynamic equilibrium corresponding to the new parameter value. Given a sufficient lag in their "perception" of the rate of price change, they may oscillate for many periods around the new dynamic equilibrium position. For some choices of parameters, the oscillations increase in amplitude until the computer program "blows up".

Fig. 8.2, p. 292, shows a typical run of the "growth" program. Note that land price, output, and distribution oscillate out of phase, with price leading.

8.6 General Dynamic Equilibrium with Many Persons

If there are $N > 2$ farmers in the economy, the results resemble those for only two in Sec. 8.4:

If per capita land in the economy $B = E/N > T_{min}$, land size at the bottom of the price "U", then only unequal distribution is stable. If landlords act as individuals, the only possible distribution is 1 landlord and $N - 1$ identical peasants. If a few landlords, N^d, act as a group, then a stable unequal distribution can exist between them and $N^p = N - N^d$ identical peasants.

If per capita land $B < T_{min}$, equal distribution is stable. But for B and N large enough, stable very unequal distributions with one or few landlords may also exist.

Multi-Person Dynamic Equilibrium Requirements:

Look at Fig. 8.6, which is simply 8.4 with a change in scale. e(T) is the dynamic equilibrium price, as before. The curves radiating from the bottom of the "U" mark the locus of per capita land size necessary for a particular distribution.

The curve marked "1/2" lies equidistant from the two sides of the "U". So, for the per capita land size at any point on this curve, there exists an unequal dynamic equilibrium distribution with the same number of landlords and peasants at the land size and price combinations at the two sides of the "U". The curve marked "1/4" lies 1/4 of the way from the left side of the "U" and 3/4 of the way from the right side of the "U". So, for the per capita land size at any point on this curve, there exists an unequal dynamic equilibrium distribution with a ratio of three peasants to one landlord. Fig. 8.6 shows curves for 1/8, 1/4, 1/3 1/2 and 3/4. However, any number of curves could be drawn, up to $N - 1$,

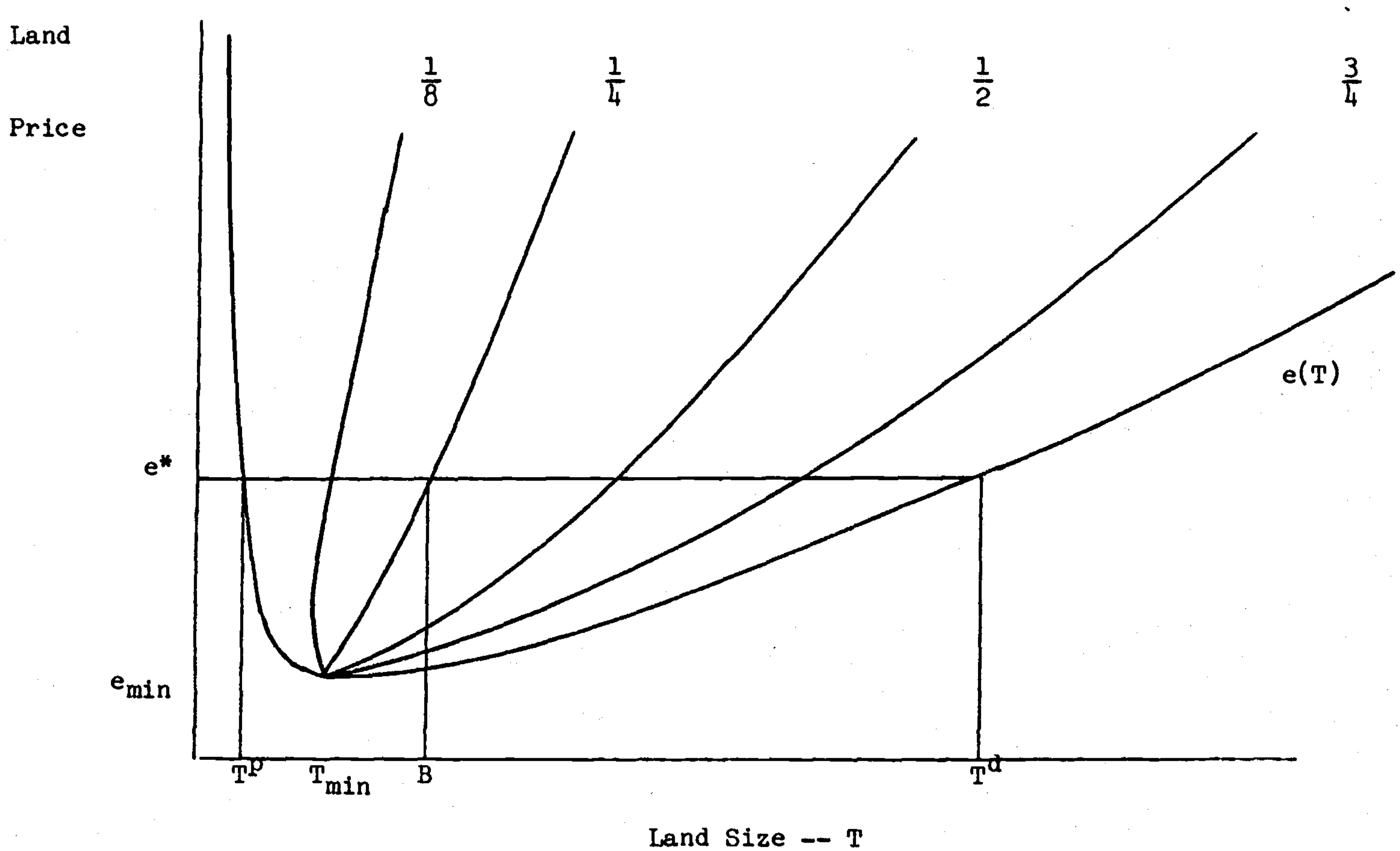


Fig. 8.6: Dynamic equilibrium no-sale price e(T) as a function of land size T. The lines radiating upwards from T_{min} mark land per capita required for unequal distribution ratios of 1 landlord in 8 persons, 1 in 4, 1 in 2, and 3 in 4. Say that per capita land equals B. Then if the landlord to total population ratio = 1/4, the size of a peasant's land and a landlord's land equals T^p and T^d respectively, and land price is e*.

where N equals the total population.

Notice that the curve marked "1/2" and all curves to the right of it slope upwards to the right. Curves to the left of the "1/2" curve slope first to the left and then turn right; the smaller the fraction, the greater the greater the leftward-sloping region. This pattern occurs because the higher the price, the greater the difference in the (absolute) slope of the price curve e(T) on opposite sides of the "U".

Fig. 8.6 quickly shows how many positions of dynamic equilibrium distribution exist for a population size, N, and a per capita land size $B = E/N$, where E is total land in the economy.

First, imagine all $N - 1$ distribution curves, marked with fractions N^d/N, from $N^d = 1$ at the left, to $N^d = N - 1$ at the right. Now imagine a vertical line at B, the land per capita. The intersections of that vertical line with the distribution curves mark all the possible dynamic equilibrium distributions for a population of N with land per capita B.

Suppose $B > T_{min}$. Then obviously, all $N - 1$ unequal distributions are possible. And the smaller the proportion of landlords in the population, the larger the gap between the peasants' land size and the landlords' land size.

But suppose $B < T_{min}$. Then only some unequal distributions are possible. No distributions can exist with $NP/N \geq 1/2$. Only those unequal distributions can exist which the vertical line from B intersects.

Recall that the distribution lines to the left of 1/2 bend first left then right. So for small enough B, given N, or small enough N, given B, the vertical line completely misses the 1/N distribution line. For if $T_{1/N}$ marks the backwards-turning point of the 1/N curve, then for $B < T_{1/N}$, or $E < NT_{1/N}$, only equal distribution can exist. Fairly apparently, from inspection of Fig. 8.6, only equal distribution can exist if total land in the economy, $E \leq 2T_{min}$ --because $NT_{1/N}$ increases as N increases.

For large enough B and N, ($B < T_{min}$) the vertical line may intersect the leftmost distribution lines twice--so a more equal dynamic equilibrium distribution may exist at a lower price and less equal one at a higher price. At the lower price distributions, the smaller the proportion of landlords, the smaller the gap between peasants' and landlords' land sizes. But at the higher price, the smaller the proportion of landlords, the larger the gap--just as for $B > T_{min}$.

Stability of Multi-Person Dynamic Equilibria:

As shown in Sec. 8.3, where dynamic equilibrium price is falling, as on the left side of the "U", any position is individually stable. Therefore, for $B = E/N \leq T_{min}$, equal distribution is a stable general dynamic equilibrium.

Also as shown in Sec. 8.3, any position on the right side of the "U" is individually unstable. Consequently, an equal general dynamic equilibrium is unstable. Moreover, the position is unstable for more than one person in an unequal general dynamic equilibrium. For say there are two landlords and lots of peasants. We take a bit of land from one landlord and give it to the other. Then the richer landlord goes on buying land, and the poorer one goes on selling land until we're left with only one landlord and one more peasant.

So to make a combined landlord and peasant unequal general dynamic equilibrium stable, there can be only one landlord--as in the two-person model of Sec. 8.4.

What about stability provided peasants and landlords act as groups?

For any unequal dynamic equilibrium, let X equal the land transferred from peasants to landlords. Then peasant land size is $B - X/N^p$, while landlord land size is $B + X/N^d$, and dynamic equilibrium price must be:

(6.1) $$e(B - X/N^p) = e(B + X/N^d)$$

To make this equilibrium stable, if landlords and peasants act as groups:

(6.2) $$-\frac{e'(B-X/N^p)}{e'(B+X/N^d)} > \frac{N^p}{N^d}$$

So ratio of the (negative) slope at the peasant's position to the slope at the landlord's position must exceed the ratio of peasants to landlords. Then, if we take a bit of land, dT, from the peasants and give it to the landlords, or vice versa, they will buy and sell themselves as groups back to their former position.

Now notice that, if $T_{N^d/N}$ marks the land size where the N^d/N distribution curve turns back to the right, then at that point:

(6.3) $$-\frac{e'(B-X/N^p)}{e'(B+X/N^d)} = \frac{N^p}{N^d}$$

Above this point, where the curve slopes rightwards, inequality (6.2) holds. Below this point, where the curve slopes leftwards, the inequality does not hold. So where the upper and lower equilibrium points exist to the left of T_{min}, only the upper points are stable when the peasants and landlords act as groups. This is as it should be given that individual distribution (and therefore group distribution) is stable to the left of T_{min}.

Clearly, any viable unequal distribution must be stable for peasants and landlords as groups, including the groups of 1 landlord and N-1 peasants. So the only individual and group stable distributions lie on the upper, rightward slopes of distribution curves, and consist of one landlord and N-1 peasants. However, a small group of landlords may not act as individuals--so viable unequal distributions may consist of small numbers of landlords and large numbers of peasants, as drawn in Fig. 8.1. Fig. 8.1 shows a small spread in size of peasant land sizes, and a large spread in landlord land sizes. For disturbances should have less effect on the individually stable peasant position than on the individually unstable landlord position.