CHAPTER 4

EFFECT OF GREATER WEALTH IN A MULTI-PERIOD ECONOMY

In the real world, save for occasional conquests and revolutions, distributions of wealth seem remarkably stable, often changing little over many decades. This stability, I believe, justifies the use of single-period models in Chapters 1 through 3, and again in Chapter 7. That is, since one period much resembles the next, a single period makes a representative slice of time.

Chapter 4 extends the basic "farmer" models of Chp. 1, with and without transactions costs, into many periods. The most immediate consequence is that, with transactions costs, the richer the individual, or larger the firm, the lower the internal discount rate and return on investment. Without transactions costs, of course, discount rate and return on investment are constant economy-wide.

Not only are real world distributions fairly stable, but they are stable despite often considerable upward and downward mobility of individuals and families within the distribution. So individual and family stability of wealth can't fully explain stability of distribution. Rather, some kind of equilibrating mechanism may be at work.

Chapter 4 shows a possible mechanism: If, with transactions costs, richer people are more future-oriented than poorer ones, this difference in time preferences can keep an unequal distribution stable. Without transactions costs, to keep unequal distribution stable, time preferences must remain constant throughout the economy.

Chapter 4 lays the basis for a dynamic model of unequal distribution in Chapter 8.

Sec. 4.1 sketches the actual models in Chapter 4. Sec. 4.2 draws some broader implications.

4.1 Summary[A]

Sec. 4.3, the consumer-investor, presents the classic problem of utility maximization over many periods, subject to a wealth constraint, and to given discount rates which may differ from one period to the next. There are no restrictions on the form of the utility function, such as separability, because such restrictions eliminate the classic result: consumer-investors choose consumption in each period so as to set their marginal rates of time substitution from one period to the next equal to one plus the given discount rate for each period.

As explained in Sec. 4.3, increasing future-orientation with wealth means that the richer a person, the higher the proportion of future to present consumption he chooses, at given discount rates. Constant time preferences mean that proportions remain constant, as wealth increases, at given discount rates.

Sec. 4.4: The farmer and his firm over time. Sec. 4.4 introduces a farming firm, which produces food from land. With transactions costs, production is subject to diminishing returns; without transactions costs there are constant returns. At the end of each period, the firm can invest or disinvest by buying or selling land at a given market price. The firm maximizes profit, taking discount rates as given, setting the price of land times discount rate equal to the marginal product of land in each period.

When the firm is combined with a consumer-investor--a farmer--discount rates in each period become endogenous, dependent on the farmer's initial land size. With transactions costs, the richer the farmer, the lower the marginal product of land. Without transaction costs, marginal product remains constant. So, since price of land times discount equals marginal product, then with transactions costs, the richer the farmer, the lower his discount rate, and firm's return on investment--at least between the present and next period. (Since the farmer can sell off land, a richer farmer may not stay richer indefinitely). Without transaction costs, there can be only one market discount rate.

Farmers can freely save or dissave by buying or selling land. But suppose none of them, rich or poor, wants to save or dissave. So distribution remains the same from one period to the next. With transactions costs, and hence diminishing returns, everyone keeps the same wealth only if richer farmers are more future-oriented than poorer ones. (Else, as will be shown in Chp. 8, richer farmers dissave, and poorer ones save, returning distribution to equality). Without transactions costs, to keep distribution unchanged, rich and poor must have identical time preferences.

Table 4.1 shows the effect of greater wealth on discount rate, wealth, income, capital turnover, and future-orientation.

4.2 Broader Implications[A]

Discount Rate, Wealth, and Firm Size:

The fall in discount rate with wealth and firm size is the other side of the balance from the rise in wage with wealth and firm size. General equilibrium, given transactions costs, is not possible without both. For instance, if poorer people and smaller firms pay or impute lower wages, and obtain a higher marginal product of land, they can bid a higher rent for any kind of land than richer people and larger firms. But, due to their lower discount rate, richer people and larger firms can still bid a higher price for the kinds of land in which they enjoy a

Table 4.1

Increased Land Size, T, in a Multi-Period Model With Stable Distribution

1. Discount rate and ROI: MP of land/land price	$r = g'(T)/c$	-
2. Consumption = Income = Profit:	$y = g(T)$	+
3. Wealth = Present value of firm: $W = V = y(1+1/r)$		+
4. Capital turnover:	$TN = y/W$	-
5. Future orientation:		+

comparative advantage, as shown in Chp. 3.

A fall in discount rate with wealth and firm size is of course a necessary consequence of "capital market failure", which economists acknowlege more often than the corresponding labor market failure. Nonetheless, some find it hard to believe that the rich and big firms really get a lower return on investment--see discussion in Chp. 6 and Chp. 9.

Wealth and Future-Orientation:

Sec. 4.4 simply assumes that, to maintain a stable unequal distribution, future-orientation rises smoothly with wealth. A more realistic assumption is that average future-orientation rises with wealth. This assumption would produce social mobility: At each level of wealth, those with greater than average (for that wealth) future-orientation would be saving and growing wealthier; while those with lower than average future-orientation would be dissaving and growing poorer. Yet overall distribution needn't change.

A rise in future-orientation with wealth exaggerates differences between richer and poorer due to transactions costs. Even without the rise, richer people would necessarily have a lower discount rate than poorer ones. The rise in future-orientation enlarges the difference in discount rates. Without transactions costs, of course, differences in future-orientation cannot affect the uniform market discount rate.

4.3 The Consumer-Investor[C]

The utility of the consumer-laborer in Sec. 1.4 depended on consumption of food, Q, and leisure, Z, in one period. Now suppose his utility depends on consumption of food, Q_o, Q_1, Q_2, ..., and leisure, Z_o, Z_1, Z_2, ... in all periods. Thus his utility becomes:

$$(3.1) \qquad U = u(Q_o, Z_o, Q_1, Z_1, Q_2, Z_2, \ldots)$$

His value of consumption equalled income in a one period model. With many periods it becomes for each period i (with food price = 1):

$$(3.2) \qquad C_i = Q_i + w_i Z_i$$

He has an endowment of wealth, W_o, at time zero. He can also exchange income from one period to the next at given prices, $1 + r_1$, $1 + r_2$ etc., where r_1, r_2 are discount rates. This wealth puts a budget constraint on him:

$$(3.3) \qquad W_o = C_o + \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_1)(1+r_2)} + \ldots + \frac{C_n}{\prod_1^n (1+r_i)} + \ldots$$

So the consumer-laborer of Sec. 1.4 becomes an investor as well. He can maximize utility subject to (3.2) and (3.3).

However, it's possible to simplify the presentation greatly with no loss of results, by: a) eliminating leisure from utility, and labor from production in Sec. 4.4 below, and b) in Sec. 4.4 below, assuming production, strictly a function of land size, shows diminishing returns to scale due implicitly to transactions costs. For, as in Sec. 1.5, the self-sufficient farmer, transactions costs make labor a function of land size, and create diminishing returns to land size.

Table 4.2

Notation for Sections 4.3 and 4.4 of Chapter 4

$0, 1, 2, \ldots, n, \ldots$ -- Periods. 0 is present.

$Q_o, Q_1, Q_2, \ldots, Q_n, \ldots$ -- Food in each period

$W_o, W_1, W_2, \ldots, W_n, \ldots$ -- Wealth in each period. W_o is initial endowment.

$C_o, C_1, C_2, \ldots, C_n, \ldots$ -- Value of consumption in each period, equals income when no saving occurs.

$u(Q_o, Q_1, Q_2, \ldots, Q_n, \ldots)$ -- Utility as a function of food

in all periods. $\frac{du}{dQ_i} \equiv u_i > 0$

$s(C_i, C_{i+1}) \equiv \frac{u_i}{u_{i+1}} = -\left.\frac{dC_{i+1}}{dC_i}\right|_u$ -- marginal rate of substitution

of consumption in one period for that in the next. It equals minus the slope of the indifference curve.

$\frac{ds}{dC_o} \equiv s_o < 0 \quad \frac{ds}{dC_1} > 0$, etc.

$v(C_o, W_1)$ -- Utility as a function of initial food and next period wealth. $\frac{dv}{dQ_o} \equiv v_o > 0 \quad \frac{dv}{dW_1} \equiv v_1 > 0$

$m(C_o, W_1) \equiv \frac{v_o}{v_1} \equiv -\left.\frac{dW_1}{dC_o}\right|_v$ -- marginal rate of substitution, equal

to minus the slope of the indifference curve, as function of initial consumption and next period wealth.

$\frac{dm}{dC_o} \equiv m_o < 0, \quad \frac{dm}{dW_1} \equiv m_1 > 0$

$r_1, r_2, r_3, \ldots, r_n, \ldots$ -- discount rates between periods 0 and 1, 1 and 2, 3 and 3, etc.

$T_o, T_1, T_2, \ldots$ -- size of land owned in each period

c -- price of land, assumed constant

$g(T_i)$ -- output of land in period i; $g'(T_i) > 0$; $g''(T_i) < 0$ ex. for constant returns to scale, when $g'' = 0$.

$V_o, V_1, V_2, \ldots$ -- present value of firm in each period, at given discount rates.

So now the consumer-investor maximizes utility:

$$(3.1)' \quad u(Q_o, Q_1, Q_2, \ldots, Q_n, \ldots)$$

subject to the wealth constraint (3.3) and the simplified consumption constraint:

$$(3.2)' \quad C_i = Q_i \qquad \forall\ i \geq 0$$

obtaining the first-order conditions:

$$(3.4) \quad \frac{\frac{du}{dQ_o}}{\frac{du}{dQ_1}} \equiv s(C_o, C_1) \equiv -\left.\frac{dC_1}{dC_o}\right|_u = 1 + r_1$$

$$\frac{\frac{du}{dQ_1}}{\frac{du}{dQ_2}} \equiv s(C_1, C_2) \equiv -\left.\frac{dC_2}{dC_1}\right|_u = 1 + r_2$$

$$\frac{\frac{du}{dQ_o}}{\frac{du}{dQ_n}} = \prod_1^n (1+r_i) \qquad \text{etc.}$$

The marginal rate of substitution of present consumption for next period consumption, $s(C_o, C_1)$, equals one plus the discount rate, r_1, and so on for every pair of periods. So consumption (of food) in every period depends on (exogenous) wealth endowment, W_o, and the exogenous discount rates, r_1, $r_2, \ldots$

Then saving in each period is the discounted value of the difference between this and next period wealth:

$$(3.5) \quad S_i = \frac{W_{i+1} - W_i}{1+r_{i+1}} = \frac{W_i \ r_{i+1}}{1 + r_{i+1}} - C_i$$

And income obviously equals:

$$(3.6) \qquad y_i = C_i + S_i = \frac{W_i \, r_{i+1}}{1 + r_{i+1}}$$

A Useful "Pseudo" Solution to the Maximization Problem:

The host of first-order equations in (3.4) are cumbersome. A "pseudo" solution to the same maximization problem is to have the consumer-investor choose between present consumption and next period wealth. This approach yields the same results much more simply, makes the graphics neater and more revealing, and permits a computer simulation in Chp. 8.

Notice that wealth in the next period, W_1, is related thus to present wealth, W_o, and consumption, C_o:

$$(3.7) \qquad W_o = C_o + \frac{W_1}{1 + r_1} \qquad \text{or} \qquad W_1 = (W_o - C_o)(1+r_1)$$

Then, since $Q_o = C_o$, utility can be written as a function of present consumption and next period wealth:

$$(3.8) \qquad v(C_o, W_1)$$

And maximization of utility subject to (3.7) yields one first-order condition:

$$(3.9) \qquad \frac{\frac{dv}{dC_o}}{\frac{dv}{dW_1}} = \frac{v_o}{v_1} = -\left.\frac{dW_1}{dC_o}\right|_v = m(C_o, W_1) = 1 + r_1$$

The marginal rate of substitution, which is minus the slope of the indifference curve between present consumption and future wealth, equals one plus the discount rate. So present consumption, C_o and next period wealth, W_1, apparently depend on the exogenous variables, present wealth W_o and discount between present and next period, r_1.

This is a pseudo solution because the results also implicitly depend on discount rates for all periods, and next period wealth, W_1, depends on utility maximization in all periods. But conceptually and graphically, the results are the same as in the correct solution.

Fig. 4.1 shows the pseudo solution.

The Marginal Rate of Substitution Functions:

The marginal rate of substitution functions, $s(C_o,C_1)$ and $m(C_o,W_1)$ are defined in (3.4) and (3.9). At constant u or v, $s(C_o,C_1)|_u$ and $m(C_o,W_1)|_v$, trace the indifference curves between present and next period consumption, and between present consumption and next period wealth. The functions have the following properities, by the assumption that consumption and wealth are normal goods:

(3.10) $s_o, m_o < 0$

(3.11) $s_1, m_1 > 0$

In other words, an increase in present consumption, holding future consumption or wealth constant, flattens the slope of the indifference curve. An increase in future consumption or wealth, holding present consumption constant, steepens the slope of the indifference curve.

Time Preferences of the Consumer-Investor as a Function of Wealth, W_o:

An individual's time preferences may remain constant as his wealth increases. Or he may becomes more future-oriented, or more present-oriented. To permit stable unequal distribution, it will be shown in Sec. 4.4 that: With transactions costs, richer individuals must be more future-oriented. Without transactions costs, time preferences must remain constant.

What do different time preferences mean, conceptually, graphically,

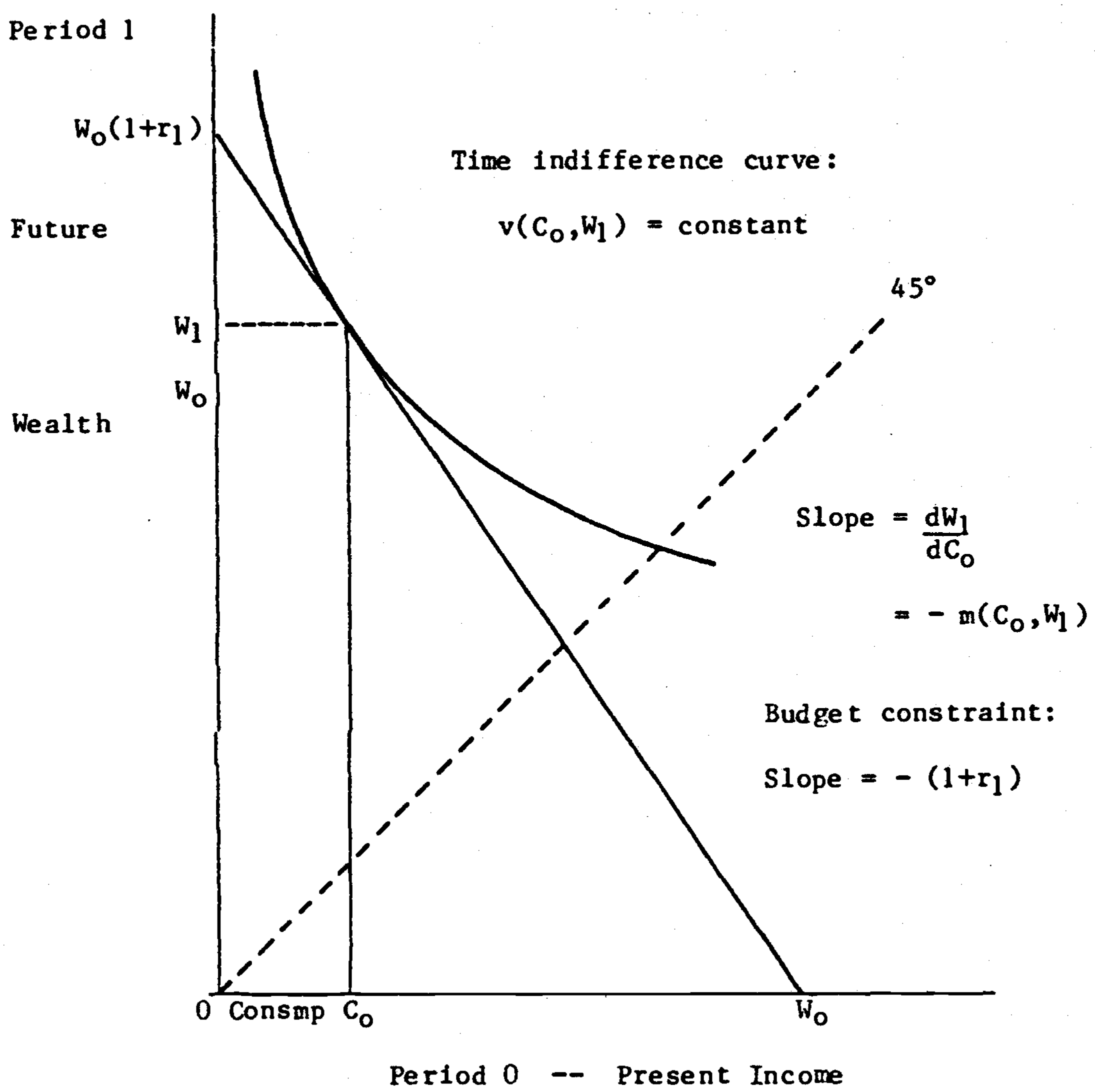

Fig. 4.1: Present consumption, C_o, and future wealth, W_1, as functions of initial wealth, W_o, and the given discount rate, r_1.

In this illustration, the consumer-investor accumulates wealth. For his future wealth, W_1, exceeds his initial wealth, W_o.

An increase in initial wealth, W_o, shifts the budget constraint out parallel to the original line, increasing both present consumption and future wealth. An increase in the discount rate, r_1, rotates the budget constraint upwards with a pivot at W_o. Future wealth obviously increases. Present consumption falls if the price effect--the greater price of present consumption relative to future wealth--outweighs the effect of greater future wealth.

and mathematically?

Constant time preferences. As the consumer-investor's wealth W_o rises, holding constant r_1 (and other discount rates), he chooses the same ratio of future consumption or wealth to present consumption. The time indifference curves of his utility functions, $u(Q_o,Q_1,\ldots)$ or $v(C_o,W_1)$, retain a constant slope along a ray from the origin, as shown in Fig. 4.2. He has a "homothetic" indifference map.

Increasing future-orientation. As the consumer-investor's wealth rises, holding discount rates constant, he chooses an increasing ratio of future consumption or wealth to present consumption. His time indifference curves flatten along a ray from the origin, as shown in Fig. 4.3.

Increasing present-orientation. As the consumer-investor's wealth rises, holding discount rates constant, he chooses a falling ratio of future consumption or wealth to present consumption. His time indifference curves steepen along a ray from the origin.

These three possibilities can be expressed formally in terms of the marginal rate of substitution functions, $s(C_o,C_1)$ and $m(C_o,W_1)$, which give the slope of the indifference curve at any point.

Let C_o and C_1 or C_o and W_1 increase in constant ratio, t:

(3.12) $C_1 = tC_o$

(3.13) $W_1 = tC_o$

Then, when C_o increases, holding t constant:

(3.14) $\left.\frac{ds}{dC_o}\right|_t = s_o + \frac{C_1}{C_o} s_1 \quad \begin{array}{ll} = 0 & \text{constant slope} \\ < 0 & \text{flattening slope} \\ > 0 & \text{steepening slope} \end{array}$

(3.15) $\left.\frac{dm}{dC_o}\right|_t = m_o + \frac{W_1}{C_o} m_1 \quad \begin{array}{ll} = 0 & \text{constant slope} \\ < 0 & \text{flattening slope} \\ > 0 & \text{steepening slope} \end{array}$

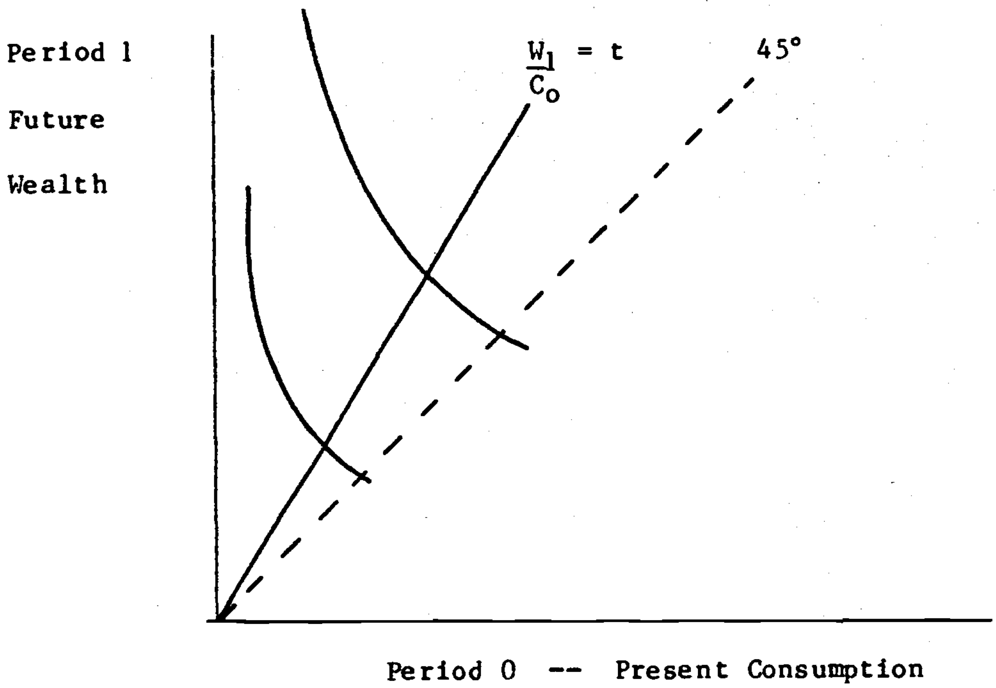

Fig. 4.2: Indifference curves between present consumption and future wealth, assuming time preferences remain constant as initial wealth increases. Slope of indifference curves remains constant on ray from origin: $W_1 = tC_o$.

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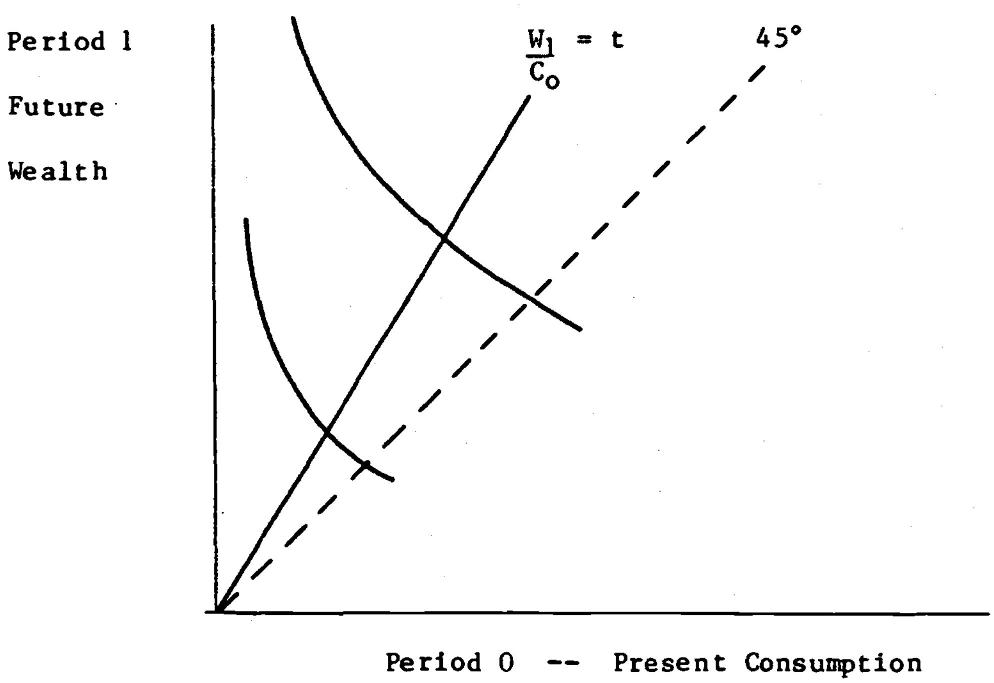

Fig. 4.3: Indifference curves between future wealth and present consumption assuming future orientation increases with initial wealth. Slope of indifference curves flattens along ray from origin, $W_1 = tC_o$.

4.4 The Farmer and His Firm Over Time[C]

As in the one-period model of the self-sufficient farmer in Sec. 1.5, suppose the consumer-investor of 4.3 owns a farming firm. The firm owns a piece of land of size T_o in Period 0, the present. Production in each period depends explicitly only on land size $g(T_i)$ where i is the period. However, production depends implicitly on labor in the following assumptions:

a. With transactions costs, production shows decreasing returns to scale, so that $g'(T_i) > 0$, and $g''(T_i) < 0$.

b. Without transactions costs, production shows constant returns to scale, so that $g'(T_i) = \text{const}$.

Production occurs "instantaneously" in each period. At the end of each period, the firm can buy or sell some land at a given market price, c. So it invests or disinvests by the amount $c(T_i - T_{i+1})$. In so doing, it determines the land available for production in the next period.

That makes cash profit in each period:

$$(4.1) \qquad P_i = g(T_i) + c(T_i - T_{i+1})$$

(Cash profit equals true profit plus investment or disinvestment).

So the present value of the firm becomes its discounted cash flow:

$$(4.2) \qquad V_o = P_o + \frac{P_1}{1 + r_1} + \frac{P_1}{(1+r_1)(1+r_2)} + \ldots + \frac{P_n}{\prod_1^n (1+r_i)} + \ldots$$

where r_1, r_2, ... are, to the firm, exogenous discount rates.

Given T_o, land price, c, and the discount rates; the firm maximizes present value, subject to (4.1), obtaining the first-order conditions:

$$(4.3) \qquad g'(T_i) - cr_i = 0 \qquad i = 1,2,3, \ldots$$

The marginal product of land in each period, starting with the next period, Period 1, equals the discount rate times the price. In other words, the marginal product of land equals the yield on the price.

Alternately:

$$(4.4) \qquad c = \frac{g'(T_i)}{r_i} \qquad i \geq 1$$

The per acre value of land equals the present value of an infinite stream of per acre yield from an increment of land.

A Useful Pseudo Solution:

The firm's maximization problem has a handy pseudo solution just like the pseudo solution to the consumer-investor's maximization. And that is to maximize present value as a function of present period profit and next period present value, V_1:

$$(4.5) \qquad V_o = P_o + \frac{V_1}{1 + r_1}$$

subject to:

$$(4.6) \qquad P_o = g(T_o) + c(T_o - T_1)$$

yielding the first-order condition:

$$(4.7) \qquad 1 + r_1 - \frac{1}{c}\frac{dV_1}{dT_1} = 0$$

so we know from the proper solution, (4.3) that:

$$(4.8) \qquad \frac{dV_1}{dT_1} = c(1 + r_1) = c + g'(T_1)$$

Present Value and Liquidation Value of Firm:

If the production function shows constant returns, ie. $g'(T_i)$ = const then there can be only one discount rate, $r = cg'$, and

(4.9) $$P_o = g'T_o + c(T_o - T_1)$$

(4.10) $$P_i = g'T_i + c(T_i - T_{i+1}) = c[(1+r)T_i - T_{i+1}] \quad i \geq 1$$

So:

(4.11) $$V_o = cT_o + g'T_o$$

The maximized present value of the firm just equals the "liquidation value" of the firm: market value of the land at given price, c, plus Period 0 production.

Diseconomies of scale in production due to transactions costs make the present value of the firm exceed the liquidation value. For

(4.12) $$P_i = g(T_i) + c(T_i - T_{i+1}) > g'(T_i)T_i + c(T_i - T_{i+1})$$

The Self-Sufficient Farmer in Many Periods: Consumer-Investor and Firm Combined:

The consumer-investor equations can be combined with the firm equations for both the real and the pseudo solutions to the maximization problem by noting that the consumer-investor's consumption in each period equals the firm's cash profit: $C_i = P_i$; and the consumer-investor's wealth in all periods equals the firm's present value: $W_i = V_i, \forall i$.

Then the combined first-order conditions for the real solution (3.4) and (4.3) are:

(4.13)
$$\frac{\frac{du}{dQ_o}}{\frac{du}{dQ_1}} \;=\; -\left.\frac{dC_1}{dC_o}\right|_u \;=\; s(C_o,C_1) \;=\; 1 + r_1 \;=\; 1 + \frac{g'(T_1)}{c}$$

$$\frac{\frac{du}{dQ_1}}{\frac{du}{dQ_2}} \;=\; -\left.\frac{dC_2}{dC_1}\right|_u \;=\; s(C_1,C_2) \;=\; 1 + r_2 \;=\; 1 + \frac{g'(T_2)}{c}$$

e tc.....

This combination renders the discount rates, r_1, r_2,... endogenous functions of the only two remaining exogenous variables, initial land size, T_o, and price of land, c.

The two pseudo solutions (3.9) and (4.7) combined yield:

(4.14)
$$\frac{\frac{dv}{dC_o}}{\frac{dv}{dW_1}} \;=\; -\left.\frac{dW_1}{dC_o}\right|_v \;=\; m(C_o,W_1) \;=\; 1 + r_1 \;=\; 1 + \frac{g'(T_1)}{c}$$

<u>Changes in Next Period Land, T_1, and Discount Rate, r_1, with Changes in Present Land, T_o:</u>

Substituting from the equation for present profit, P_o, (4.6); (4.14) can be rewritten:

(4.15)
$$m[g(T_o)+c(T_o-T_1),W_1] - 1 - \frac{g'(T_1)}{c} \;=\; 0$$

Derivatives with respect to T_o then show what happens to next period land and discount rate with an increase in price of present land. Using (4.8) that $\frac{dW_1}{dT_1} = \frac{dV_1}{dT_1} = c + g'(T_1) > 0$ it follows that:

A richer farmer in the present is also a richer farmer in the next period. That is, if we increase a farmer's initial land, T_o, he does not immediately sell off the entire increase, as apparent from:

(4.16) $$\frac{dT_1}{dT_o} = - \frac{m_o(g'(T_o) + c)}{- m_oc + m_1(g'(T_1)+c) - \frac{g''(T_1)}{c}} > 0$$

Therefore, --except in the absence of transaction costs, when $g'(T) = \text{const.}$, --discount rate for the next period falls with initial land size:

(4.17) $$\frac{dr_1}{dT_o} = \frac{g''(T_1)}{c}\frac{dT_1}{dT_o} < 0$$

So the richer the farmer, given transactions costs, the lower his discount rate. And the larger the firm, the lower its return on investment.

Constant Distribution of Wealth in Each Period:

Suppose that at the market price of land, c, none of the farmers, rich, poor, or middling, wishes to save and invest. That is, none of the farmers want to buy or sell land at price c at the end of each period. So distribution of wealth remains constant from one period to the next.

This is possible only if: A) there are no transactions costs and hence production shows constant returns to scale, so $g'(T) = \text{const.}$ or, B) richer farmers have more future-oriented time preferences as described at the end of Sec. 4.3.

For without saving and investment, profit, consumption, and income are the same for each period, and wealth in all periods equals present value of income (dropping subscripts):

(4.18) $$P = C = y = g(T)$$

(4.19) $$W = V = (1 + \frac{1}{r})\, y$$

$$(4.20) \quad s(y,y) = m(y,W) = 1 + r = \frac{1}{c}\frac{dW}{dT} = 1 + \frac{g'(T)}{c}$$

$$(4.21) \quad \frac{dr}{dT} = \frac{g''(T)}{c} < 0 \quad \text{ex. for const. returns, as in (4.18)}$$

Consequently capital turnover, y/W, also falls with wealth, except for constant returns.

From (3.14) and (3.15) the conditions on time preferences become, for the real and pseudo solutions respectively:

$$(4.22) \quad \left.\frac{ds}{dy}\right|_t = s_o + s_1 \quad \begin{matrix} = 0 & \text{constant time preferences} \\ < 0 & \text{increasing future orientation} \end{matrix}$$

$$(4.23) \quad \left.\frac{dm}{dy}\right|_t = m_o + (1+\frac{1}{r})\, m_1 \quad \begin{matrix} = 0 & \text{constant time preferences} \\ < 0 & \text{increasing future-orientation} \end{matrix}$$

--where t is, respectively, a constant ratio of next period to this period income, or next period wealth to present period income.

From (4.17), $s(y,y) = 1 + r$. So from (4.22) it follows that: If r is constant, for constant returns to scale in the absence of transactions costs, then $s_o + s_1 = 0$, and time preferences are constant too. If r falls as wealth increases, because transactions costs create diminishing returns, then $s_o + s_1 < 0$, and future-orientation increases with wealth.

(It's possible to show the same thing using (4.23), by writing (4.16) with the condition that $\frac{dT_1}{dT_o} = 1$, and $T_o = T_1$).

Figure 4.4 shows the pseudo solution to a farmer's maximization problem, under the constraint that distribution remains constant.

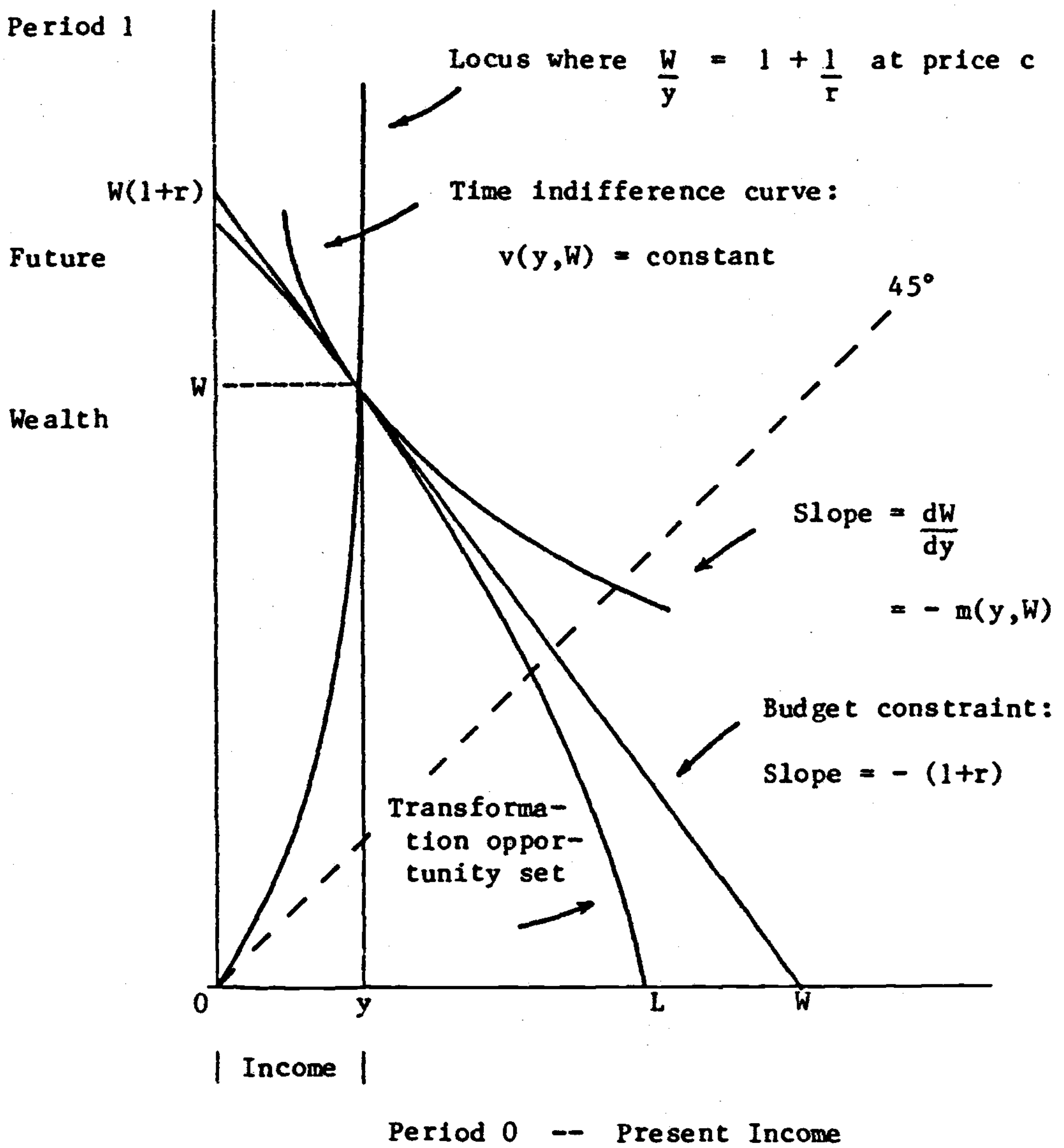

Fig. 4.4: The farmer maximizes utility at the point his time indifference curve lies tangent to his budget constraint and his transformation opportunity set.

Assume land size, T, and therefore W, y, and r remain constant through time. The locus where $W/y = 1 + 1/r$ marks the combinations of W and y that meet this condition at land price c.

$L = g(T) + cT < W$ is the liquidation value of the firm.