CHAPTER 3

WEALTH AND COMPARATIVE ADVANTAGE; INDUSTRY SPECIALIZATION; LOCATION; SOCIAL CLASS; MONOPOLY

In the real world richer and poorer people quite strikingly cluster together by occupation, place of residence, place of work, type of assets owned, including type of business owned. In particular, richer people tend to own (or own stock in) and work for larger businesses.

Likewise, in the real world larger and smaller companies cluster by industry and location. Larger companies tend to less labor-intensive industries, offering greater economies of scale, and often (but not always) allowing significant market power.

Transactions-cost-less economics explains clustering by wealth or firm size, if at all, by allusions to "tastes" or "economies of scale". But allowing transactions costs, such clustering follows immediately and rigorously from the venerable principle of comparative advantage. That is, richer people's and bigger companies' greater size of assets and higher internal ratio of capital to labor gives them a comparative advantage in activities with greater economies of scale or lower intrinsic labor intensity, and therefore in owning land or other assets most suited to those activities. *Mutatis mutandis* for poorer people and smaller companies.

Sec. 3.1 summarizes what is actually shown in Chp. 3. Sec. 3.2 draws some larger implications.

3.1 Summary[A]

3.3 Wealth and Comparative Advantage:

Define intrinsic labor intensity as follows: Suppose there are two production functions that depend on quantity of land and labor. Suppose there is a range over which, holding the marginal product of labor equal for both functions, the labor share of output is higher for the first function than for the second. (Since labor share equals marginal product divided by average product of labor, average product is therefore necessarily lower for the first than for the second over the same range). Then the function with the higher labor share (and lower average product of labor) is intrinsically more labor-intensive in that range.

Intrinsic labor intensity is a built-in property of production functions, to be distinguished from the fact that it's possible to carry on any particular form of production less or more labor-intensively by applying less or more labor to a given area of land. Low intrinsic labor intensity corresponds loosely to what we consider "high quality" in a resource or production process. For example, more fertile soil grows more vegetables with less effort.

Sec. 3.3 shows that, given transactions costs, larger landowners have a comparative advantage in production that is intrinsically less labor-intensive and/or shows greater economies of scale. This comparative advantage implies a geographical sorting by wealth. Richer landowners own land better suited to production that is intrinsically less labor-intensive or shows greater economies of scale, while poorer landowners own land better suited to production with the opposite characteristics. Comparative advantage also implies that the "highest and best use" of a particular piece of land may depend on the wealth of the owner.

3.4 Unequal Wealth and Classical Location Theory:

Classical location theory as originated by Von Thünen posits a market town located in a "featureless plain". Transportation costs depend solely on radial distance from market. Then economic activities with a relatively high per acre profit and high unit transportation cost enjoy a comparative advantage in more central locations, while those with a relatively low profit and low unit transportation cost enjoy a comparative advantage in more peripheral locations. Consequently, activities arrange themselves in a bullseye pattern around the center, with the highest per acre profit and transportation cost activity closest to market, and the lowest farthest from market. For example, in the model of 3.4, there is a ring of fruit trees around town, with a ring of grain around that, and a ring of horse pasture on the outside.

Wealth differences fit into this classic model as follows:

Assuming transportation is primarily a labor cost, then the intrinsic labor intensity of any production in any location depends both upon the natural (transportation cost-less) intrinsic labor intensity of the "highest and best use" activity in that location, and upon distance from the center. The lower the unit transportation cost, the less transportation adds to intrinsic labor intensity. Consequently, 1), intrinsic labor intensity of production rises from the inner to the outer edge of each activity. And 2), intrinsic labor intensity then drops abruptly at the boundary where one activity gives way to the next, lower transportation cost activity, (unless the next activity has much greater natural labor intensity, in which case intrinsic labor intensity may jump up at the boundary). So in net, intrinsic labor intensity (usually) forms a sawtooth curve moving out from the center.

From 3.3, transactions costs give larger landowners a comparative advantage in intrinsically less labor-intensive production. As a result, the richest landowners occupy the inside of each activity ring--that is, the best fruit land, the best grain land, and the best horse land--with a gradual decline in wealth with distance from the center, and then (usually) an abrupt increase at the next activity.

Moreover, at the fruit-grain and grain-horse boundaries, there are belts of land where highest and best use depends on the wealth of the landowner.

Similar results would hold if land varied continuously in qualities other than access to market. For example, suppose than on "more fertile soil", or in a "richer mine", the same area of land and hours of labor yield greater output. Production on such land is intrinsically less labor intensive. Then the largest landowners would occupy the most fertile soil or richest mines, and wealth would fall continuously as soil or mine quality fell.

3.5 Wealth and Supervision Costs:

3.5 shows that, all else being equal, production with relatively high supervision requirements is intrinsically more labor-intensive. Consequently, from 3.3 it follows that larger landowners have a comparative advantage in relatively low supervision requirement production.

This result implies that richer people and bigger firms should prefer more routinized and easily monitored kinds of activities, to save on scarce managerial time.

3.2 Broader Implications of Comparative Advantage[A]

Behavior of Richer and Poorer, Larger and Smaller Firms:

The paradigm model of the self-sufficient farmer in Sec. 1.5 predicts a variety of differences between richer and poorer, larger and smaller, assuming they all occupy the same quality land, engaged in the same production. Some of these differences hold up when land quality and production may vary; others do not.

Richer people and larger firms both have a comparative advantage in intrinsically less labor-intensive production and carry on any given production in a less labor-intensive fashion, that is, using a lower ratio of labor to land. So comparative advantage reinforces the prediction that output per manhour rises with wealth and firm size.

By contrast, the prediction of the self-sufficient farmer model that richer people and larger firms obtain lower output per acre--holds only where richer and poorer, larger and smaller, engage in the same production on the same quality land. Less intrinsically labor-intensive production, on better quality land, may yield higher output per acre than more intrinsically labor-intensive production on lower quality land, as shown in Sec. 3.4. A preference for low intrinsic labor intensity may outweigh the tendency of richer people and larger firms to obtain lower output per acre, obscuring their relatively light use of resources.

Social Class:

In the models of this chapter, comparative advantage makes persons of different wealth choose to own different kinds of land applied to different kinds of activities. So, implicitly, wealth determines occupation. However, the same argument that applies to choice of land can apply directly to choice of human capital investments: a person's wealth endowment (cash and physical asset inheritance as well as skills taught by family or school) determines his or her comparative advantage in choosing and training for an occupation. Thus a poor person has a comparative advantage in unskilled labor, while a rich one has a comparative advantage in lawyering. Chapter 14 discusses this point at length.

Chapter 2 showed that persons tend to restrict their hiring and renting to other persons of similar wealth. This tendency causes some economic clustering, along a continuum with no distinct levels. However, clustering due to comparative advantage in occupation and location can break up the spectrum of wealth into discreet levels. The population may appear stratified into clear "social classes", distinguished simultaneously by wealth, education, occupation, place of residence, place of work, and preferred types of physical and financial assets.

Monopoly and Comparative Advantage:

Both conventional theory and practical experience predict that activities offering large economies of scale invite monopoly. The conventional argument, which implicitly assumes transactions costs, follows from the explanation of operating firm size in Chp. 2: Holding constant wealth and ability of owners, the greater the underlying economies of scale in technology, the larger the average operating size of firms in an industry. The larger the average operating size of firms, the fewer the market can support, making it more likely that each has substantial market power. And then of course there are economies of scale in the exercise of market power, further increasing operating size.

But this conventional argument in isolation suggests that large firms in concentrated industries should be more leveraged--the opposite of reality.

Comparative advantage comes to the rescue, predicting that:

Not only do economies of scale lead to greater operating sizes of firms for owners of a given wealth and ability, they also attract previously wealthier owners, or companies that have already accumulated a larger mass of assets. So economies of scale in technology increase not only operating size, but also size of equity ownership of firms. (Of course, historically, the line of causation often runs the other way: firms that first exploit a new technology offering great economies of scale may get very large, and their owners very rich).

Then, if owners and managers of large firms come from the same economic and social background, they may find it easier to reach and maintain "gentlemen's agreements" to restrict output. Or they can more easily influence government to restrict output for them.

Comparative advantage also predicts the same about industries of low intrinsic labor intensity. That is, such industries attract large firms and wealthy owners. Hence, again, a few firms may end up dominating the industry, or simply controlling the best quality resources suited to that industry (eg. the best ore reserves), without any prior intent to monopolize. So in low intrinsic labor intensity may lie the explanation of concentration in industries, like oil and autos, that don't seem to offer any overwhelming technological economies of scale.

Thus large firm size and industry concentration may result more from underlying concentration in the ownership and control of wealth, than from technology. Technology--economies of scale and low intrinsic labor intensity--just explains <u>which</u> industries attract big firms and become concentrated. Were wealth more equally distributed, firms wouldn't get so big or industries so concentrated--a fact that is perfectly obvious in less developed countries where large firms belong to identifiable families. But even in the US, it's become a commonplace that, not technological economies, but "economies of scale in access to capital" account for the size of large firms. [eg. see Williamson, 1975]. And that wouldn't be possible without transactions costs.

Location and Land Use:

Real life land use patterns give but a very blurred reflection of the orderly succession of uses predicted by classical location theory. In fact, they give but a blurred reflection of the pattern as modified by transactions costs between richer and poorer: with richer landowners occupying the more centrally located or better quality parts of each activity region. In the real world, the boundaries between activities are ragged and ill-defined, in the U.S. most visibly so at the urban "fringe", where housing may "sprawl" for miles into farmland.

Another kind of transactions costs explain this raggedness: costs that hinder transfers of land between individuals. So persons of widely disparate wealth may at least temporarily own intermingled property, especially in a zone of transition from one use to another. In such a zone, as shown in Sec. 3.4, highest and best use of land depends on the owner's wealth. So transactions costs predict, instead of the sharp boundary of classical location theory, a mixing of uses in a zone of transition, generally with richer landowners in the lower-intensity, more peripheral use, and poorer ones in the higher-intensity, more central use. For example at the urban fringe in the U.S., single family housing developments typically "leapfrog" among tracts held by large speculators, hobby farms of the rich, etc.

3.3 Wealth and Comparative Advantage[C]

This section shows, loosely, that transactions costs give richer people and bigger firms a comparative advantage in production offering greater economies of scale, and/or lower intrinsic labor intensity. More precisely, the section shows:

1) Suppose there are two linear homogeneous production functions of land and labor. Then a richer farmer has a comparative advantage in production with lower intrinsic labor intensity, defined in Sec. 3.1.

2) Suppose there are two homogeneous production functions of land and labor, and suppose total elasticity of output is close to 1, and/or wages are low. Then a richer farmer has a comparative advantage in production with equal or greater total elasticity of output, and with equal or lower intrinsic labor intensity--the inequality holding in at least one case.

3) Suppose there are two arbitrary production functions of land and labor. Then comparative advantage depends not only on intrinsic labor intensity and total elasticity of output, but also on rates of change of intrinsic labor intensity and total elasticity. However, all else being equal, a richer farmer enjoys a comparative advantage in production where total elasticity of output increases faster, or decreases more slowly.

These results hold in two possible circumstances:

A. There are two different kinds of land in the economy, each with its own particular production function. In this case if richer farmers enjoy a comparative advantage in one kind of production, they will selectively occupy that kind of land, leading to geographic sorting by wealth.

B. Richer and poorer farmers occupy identical quality land, which can be (and is) used in two different forms of production. In this case, there is no unique highest and best use of land; highest and best use depends on wealth of the owner. B is really a special case of A, as will appear.

A. Comparative Advantage for Two Different Kinds of Land:

Suppose there are two kinds of land in the Clone economy. The two kinds of land differ enough that the highest and best use of one is always fruit, and of the other grain--regardless of the landowner's wealth. Then comparative advantage is determined as described in points 1, 2 and 3, above. That is, for linear homogeneous production, richer farmers have a comparative advantage in intrinsically less labor-intensive production, and so forth.

Points 1, 2, and 3 can be shown as follows:

Suppose, for simplicity, exactly half the Clone territory is fruit land and half is grain land. (Any other arbitrary ratio of fruit to grain land would do just as well). All the inhabitants are self-sufficient farmers as in Sec. 1.5. At the start, each farm, large or small, is half fruit and half grain land. Then if richer farmers can trade some of their grain land to poorer farmers in exchange for more fruit land, leaving both better off, that means richer farmers have a comparative advantage in fruit growing, and poorer farmers have a comparative advantage in grain growing.

If we let the farmers actually trade land, we expect that after trade, the richest farmers all grow fruit and the poorest all grow grain. Middle-sized farmers may grow some of each. If there are diseconomies of scale in fruit and grain production, we would expect a large range of middle-sized farmers to grow both fruit and grain. If there are economies of scale, we would expect farmers up to a certain size to grow only grain, and farmers over that size to grow only fruit.

A necessary and sufficient condition for the beneficial exchange of land between richer and poorer farmers, as shown in Appendix 1, Sec. 3.6, is that the ratio of marginal products of fruit and grain land be higher for richer than for poorer farmers. In fact, suppose r^d is the ratio for richer farmers and r^p is the ratio for poorer farmers, and $r^d > r^p$. Then any ratio of exchange of grain for fruitland, R = x grainland/x fruitland, leaves at least one farmer better off and the other no worse off, iff:

(3.1) $r^d \geq R \geq r^p$ (with at least one inequality)

So to discover what characteristics of production give richer farmers a comparative advantage in fruit, we need only find out what characteristics make the ratio of marginal products of fruit to grain land rise as a given farmer gets richer.

B. Comparative Advantage for Two Production Functions on the Same Land:

It's easy to show now that B is a special case of A, so whatever characteristics determine comparative advantage for two different kinds of land, also determine it for two production functions on the same kind of land--assuming both forms of production actually occur.

Suppose all land in the Clone economy is identical and can grow either fruit or grain. Suppose we also know that if the farmers choose their crops freely, half the land will end up in fruit and half in grain, but we don't know who will grow which.

So we make each farmer, large and small, plant exactly half his land to fruit and half to grain. Then we let farmers exchange land growing fruit for land growing grain on a 1 to 1 basis, that is, requiring that R in (3.1) equals 1. Now if it's more profitable for rich farmers to grow fruit, then their ratio of marginal products of fruit to grain land, r^d must be > 1. And if it's more profitable for poor farmers to grow grain, then their ratio $r^p < 1$. So $r^d > 1 > r^p$, and mutually beneficial trade can occur, with richer farmers trading land with grain for more land with fruit.

This little experiment shows we can find the characteristics determining comparative advantage for two functions on the same kind of land by the same procedure as for two functions on two different kinds of land: We give a farmer a piece of land, require him to grow fruit on one half and grain on the other, and see what happens to the ratio of marginal products of fruit and grain as we increase his land size. Only in this case, for richer farmers to have a comparative advantage in fruit, the ratio of marginal products must not only rise as wealth increases, but it must rise from below 1 to above 1 so that both fruit and grain can be profitable on the same land.

Characteristics of Production Determining Comparative Advantage:

Suppose a farmer owns T acres each of fruit land and grain land. To discover what characteristics of production determine comparative advantage, we will find out what characteristics make the ratio of marginal products of fruit and grain land rise as we increase T.

Define the following:

L_F -- labor applied to Fruit-land

L_G -- labor applied to Grain-land

$f(T,L_F)$ -- output of Fruit-land

$g(T,L_G)$ -- output of Grain-land

w -- farmer's wage

$B_F = f_2L_F/f$ -- labor share of output on Fruit-land

$B_G = g_2L_G/g$ -- labor share of output on Grain-land

$N_F = f_1T/f$ -- land share of output on Fruit-land

$N_G = g_1T/g$ -- land share of output on Grain-land

$e_F = B_F + N_F$ -- total elasticity of output on Fruit-land

$e_G = B_G + N_G$ -- total elasticity of output on Grain-land

$e = 1$ -- constant returns to scale
> 1 -- increasing returns to scale
< 1 -- decreasing returns to scale

The farmer's firm maximizes profit:

$$P = f(T,L_F) + g(T,L_G) - w(L_F + L_G) \tag{3.2}$$

obtaining Kuhn-Tucker conditions:

$$w - f_2 \geq 0 \qquad (w - f_2)L_F = 0 \tag{3.3}$$

$$w - g_2 \geq 0 \qquad (w - g_2)L_G = 0 \tag{3.4}$$

Assuming the farmer does in fact work on both fruit and grain land, the marginal product of labor on both kinds of land equals his wage.

Then the farmer's labor supply equation ($L = L_F + L_G = a(P+wD,w)$) makes his wage and labor supply endogenous functions of his wealth, measured by the size, T, of his fruit land and grain land. From Sec. 1.5, his wage rises with wealth (the exact expression gets messy with two kinds of land):

(3.5) $$\frac{dw}{dT} > 0$$

In fact, as shown in Sec. 1.5, his wage rises in an S-curve, from w = 0 at T = 0. So for very small land and very large land, dw/dT is close to 0.

An increase in land size, T, affects the ratio of marginal products of Fruit-land to Grain-land directly, and indirectly by increasing the farmer's wage:

(3.6) $$\frac{d}{dT}\left(\frac{f_1}{g_1}\right) = \frac{f_1}{g_1}\left[\frac{f_{11} - \frac{(f_{12})^2}{f_{22}}}{f_1} - \frac{g_{11} - \frac{(g_{12})^2}{g_{22}}}{g_1}\right] \quad \text{1}$$

$$+ \frac{f_1}{g_1}\left[\frac{\frac{f_{12}}{f_{22}}}{f_1} - \frac{\frac{g_{12}}{g_{22}}}{g_1}\right]\frac{dw}{dT} \quad \text{2}$$

Both terms 1 and 2 must be ≥ 0, or the positive one must dominate the negative one, for the ratio of marginal product of F-land to G-land to rise.

<u>The Linear Homogeneous Case:</u>

If both production functions are linear homogeneous, then the first term above = 0, as apparent from (7.29) in Appendix 2, Sec. 3.7.

From (7.30), the expression becomes simply (where B_F and B_G are fruit and grain labor shares of output when $w = f_2 = g_2$):

(3.7) $$\frac{d}{dT}\left(\frac{f_1}{g_1}\right) = \frac{f_1}{g_1}\left[-\frac{B_F}{1 - B_F} + \frac{B_G}{1 - B_G}\right]\frac{1}{w}\frac{dw}{dT}$$

$$> 0 \quad \text{iff} \quad B_F < B_G$$

But $B_F < B_G$ when $f_2 = g_2$ means fruit growing is intrinsically less labor-intensive than grain-growing. That is, the labor share of output on Fruit-land is lower than that on Grain-land--and necessarily the average product of labor on Fruit-land exceeds that on Grain-land. So for linear homogeneous functions, richer farmers have a comparative advantage in fruit if and only if fruit is intrinsically less labor-intensive than grain.

The Homogeneous Case:

For a homogeneous function, output elasticity, e, remains constant everywhere.

As shown in Appendix 2, for e close to 1, or small w, the expressions in (3.6) can be approximated, from (7.24) and (7.27):

(3.8) $$\frac{d}{dT}\left(\frac{f_1}{g_1}\right) = \frac{f_1}{g_1}\left[\frac{e_F(e_F-1)}{e_F-B_F} - \frac{e_G(e_G-1)}{e_G-B_G}\right]\frac{1}{T} + \frac{f_1}{g_1}\left[-\frac{B_F}{e_F-B_F} + \frac{B_G}{e_G-B_G}\right]\frac{1}{w}\frac{dw}{dT}$$

$$> 0 \quad \text{if} \quad B_F \leq B_G \quad \text{and} \quad e_F \geq e_G \quad \text{(one a strict inequality)}$$

-- provided that $e_G \leq 1$.

To remove this restriction, recall that while there may be economies of scale in the underlying production, diseconomies due to the scarcity of the owner's labor must dominate. Therefore:

(3.9) $$\frac{df_1}{dT} = f_{11} - \frac{(f_{12})^2}{f_{22}} + \frac{f_{12}}{f_{22}}\frac{dw}{dT} < 0$$

So from (7.24) and (7.24) again:

(3.10) $$\frac{T}{w}\frac{dw}{dT} > \frac{e_F(e_F-1)}{B}$$

(3.8) together with this inequality show that:

$$(3.11)\qquad \frac{d}{dT}\left(\frac{f_1}{g_1}\right) > \frac{f_1}{g_1}\,\frac{1}{e_G - B_G}\left[-e_G(e_G-1) + e_F(e_F-1)\frac{B_G}{B_F}\right]$$

$$> 1 \quad \text{for} \quad \left.\begin{array}{l} e_F \geq e_G \ (>1) \\ B_F \leq B_G \end{array}\right\} \text{one a strict inequality}$$

So in general:

$$(3.12)\qquad \frac{d}{dT}\left(\frac{f_1}{g_1}\right) > 0 \quad \text{for} \quad \left.\begin{array}{l} e_F \geq e_G \\ B_F \leq B_G \end{array}\right\} \text{one a strict inequality}$$

The same approximations hold for functions close to homogeneous, so that rates of change in total elasticity of output can be ignored.

So in this case, richer farmers have a comparative advantage in fruit if fruit shows greater or equal total elasticity of output, and less or equal intrinsic labor intensity, with inequality in at least one case.

The General Case:

From (7.9) and (7.13) in Appendix 2, Sec. 3.7, (3.6) can be written most generally:

$$(3.13)\quad \frac{d}{dT}\left(\frac{f_1}{g_1}\right) = \frac{f_1}{g_1}\left[\ \frac{1}{e_F-B_F}\left(\left.\frac{de_F}{dT}\right|_w + \frac{e_F-1}{f}\left.\frac{df}{dT}\right|_w\right) - \frac{1}{e_G-B_G}\left(\left.\frac{de_G}{dT}\right|_w + \frac{e_G-1}{g}\left.\frac{dg}{dT}\right|_w\right)\ \right]$$

$$+ \frac{f_1}{g_1}\left[\ \frac{B_F}{e_F-B_F}\left(\left.\frac{de_F}{dw}\right|_T - \frac{1}{w}\right) + \frac{e_F-1}{e_F-B_F}\frac{1}{f}\left.\frac{df}{dw}\right|_T - \frac{B_G}{e_G-B_G}\left(\left.\frac{de_G}{dw}\right|_T - \frac{1}{w}\right) - \frac{e_G-1}{e_G-B_G}\frac{1}{g}\left.\frac{dg}{dw}\right|_T\ \right]\frac{dw}{dT}$$

Alternately, from (7.10) and (7.14):

$$(3.14)\quad \frac{d}{dT}\left(\frac{f_1}{g_1}\right) = \frac{f_1}{g_1}\left[\frac{1}{e_F-B_F}\left.\frac{de_F}{dT}\right|_w + \frac{e_F-1}{(e_F-B_F)(1-B_F)}\left.\frac{dB_F}{dT}\right|_w + \frac{e_F-1}{1-B_F}\frac{1}{T} - \frac{1}{e_G-B_G}\left.\frac{de_G}{dT}\right|_w - \frac{e_G-1}{(e_G-B_G)(1-B_G)}\left.\frac{dB_G}{dT}\right|_w - \frac{e_G-1}{1-B_G}\frac{1}{T}\right]$$

$$+ \frac{f_1}{g_1}\left[\frac{1}{e_F-B_F}\left.\frac{de_F}{dw}\right|_T + \frac{e_F-1}{(e_F-B_F)(1-B_F)}\left.\frac{dB_F}{dw}\right|_T - \frac{B_F}{1-B_F}\frac{1}{w} - \frac{1}{e_G-B_G}\left.\frac{de_G}{dw}\right|_T - \frac{e_G-1}{(e_G-B_G)(1-B_G)}\left.\frac{dB_G}{dw}\right|_T + \frac{B_G}{1-B_G}\frac{1}{w}\right]\frac{dw}{dT}$$

(3.13) and (3.14) show how the change in the ratio of marginal products depends on rate of change of elasticity and proportional change of output or change in labor share, B. With no restrictions on the functions, the relationships can become quite complicated.

But note in particular the rate of change of total elasticity, $\left.\frac{de}{dT}\right|_w$ and $\left.\frac{de}{dw}\right|_T$. All else being equal, the ratio of marginal products of fruit and grain rises if total elasticity falls more slowly on fruit than on grain land. (e must fall with scale for any reasonable function).

3.4 Wealth and Location[C]

A. As described in Sec. 3.1, comparative advantage implies that, in the bullseye pattern of classical location theory, richer farmers occupy the inside of each activity ring. This result can be proved simply by showing that, including transportation to market, intrinsic labor intensity of production is lower on the inside of each activity ring. Moreover, there will be regions between activity rings where highest and best use depends on the wealth of the landowner.

B. It's also shown that, if transportation cost is proportional to output, then in a perfect market, output per acre falls with distance from the center. This result, combined with the predictions of comparative advantage, means location has two opposing effects on intensity of land use: Absent transactions costs (except transportation) more central land should be more intensively used, but with transactions costs, such land attracts wealthier owners, who tend to use land less intensively. The net effect can't be predicted.

The models of A. and B. both assume no transactions costs apart from labor costs of transportation to a central market. The results for a world with transactions costs then follow immediately from combining the predictions of these models with the predictions of comparative advantage derived in Sec. 3.3.

A. Effect of Transportation on Intrinsic Labor Intensity in the Classic Location Theory Model:

Assume the "featureless plain" of classical location theory (eg. see [Alonso, 1964], Chp. 3). Except for radial distance from a central market town, land quality is uniform. The land can produce three crops, fruit, grain, and horses, which must be carried to market. Production for all three crops is linear homogeneous in land and labor. Labor is available to landowners at a market wage, w. Per acre transportation labor is proportional to the distance from the center, R, and to a "carrying" coefficient, c, different for fruit, grain, or horses.

Assume further that at the market wage, the marginal product of land in fruit exceeds the marginal product of land in grain, which exceeds the marginal product of land in horses. But it costs the most per acre to carry fruit to market, less to carry grain to market, and least to drive horses to market.

Obviously, the farmers choose whichever crop maximizes per acre profit in their location. The resulting land use pattern can be found using the following notation:

w -- given "market" wage.

R -- distance to central place (radius of circle).

c_F, c_G, c_H -- transportation cost coefficient for fruit, grain and horses. Assume $c_F > c_G > c_H$.

c_FR, c_GR, c_HR -- transportation labor requirement per acre for fruit, grain, and horses.

A_F, A_G, A_H -- applied labor for fruit, grain, and horses.

$f(T,A_F)$, $g(T,A_G)$, $h(T,A_H)$ -- output as a (linear homogeneous) function of land and applied labor for fruit, grain, and horses. Assume when $w = f_2 = g_2 = h_2$, then $f_1 > g_1 > h_1$.

The Maximization Problem:

At distance R from town, with market wage w, per acre profit for each crop is:

$$(4.1)\qquad \frac{P_F}{T} = \frac{f(T,A_F) - w(A_F + c_F RT)}{T} = f_1 - wc_F R$$

$$(4.2)\qquad \frac{P_G}{T} = \frac{g(T,A_G) - w(A_G + c_G RT)}{T} = g_1 - wc_G R$$

$$(4.3)\qquad \frac{P_H}{T} = \frac{h(T,A_H) - w(A_H + c_H RT)}{T} = h_1 - wc_H R$$

Since wage w is constant, and production is linear homogeneous, f_1, g_1, and h_1 are constant. So per acre profit for each crop falls in a straight line with distance R from town. As drawn in Fig. 3.1, fruit is most profitable next to town, farther out grain becomes most profitable, and yet farther out, horses. Hence the classic bullseye pattern of activities, with fruit on the inside, then a ring of grain, and then a ring of horses.

Location and Intrinsic Labor Intensity:

Meanwhile, what happens to intrinsic labor intensity as a function of distance from town? Intrinsic labor intensity at any location has two components: "Natural" intrinsic labor intensity of the crop, not including transportation costs, plus additional intrinsic labor intensity due to transportation costs. So to show the relation between intrinsic labor intensity and distance from town, define:

$B_{Fo} = \frac{wA_F}{f(T,A_F)}$ -- labor share of output for fruit at wage w, not including transportation cost.

$B_F = \frac{w(A_F + c_F RT)}{f(T,A_F)} = B_{Fo} + \frac{wTc_F R}{f}$ -- labor share of output for fruit at wage w, including transportation cost. Note that f/T is constant given w, so B_F rises linearly with R.

Fig. 3.1

Profit per Acre

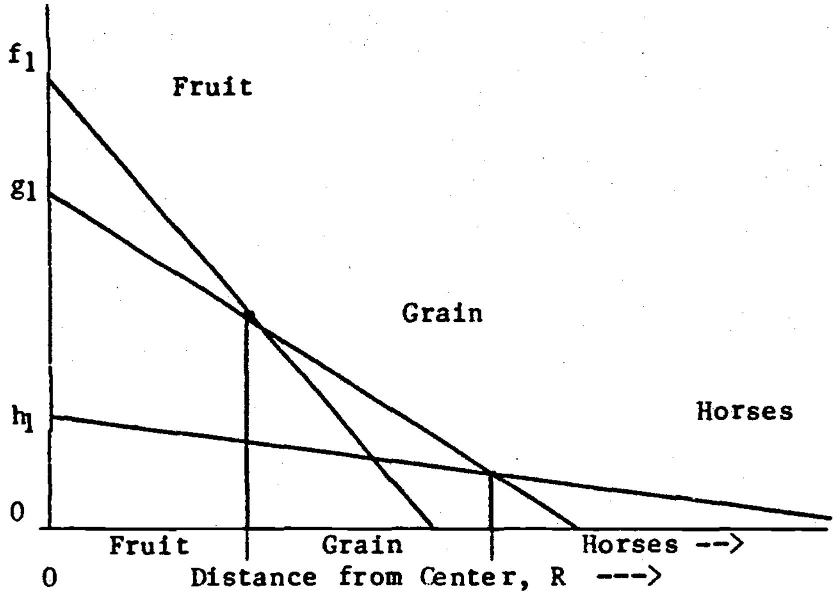

Fig. 3.2

Labor Share of Output

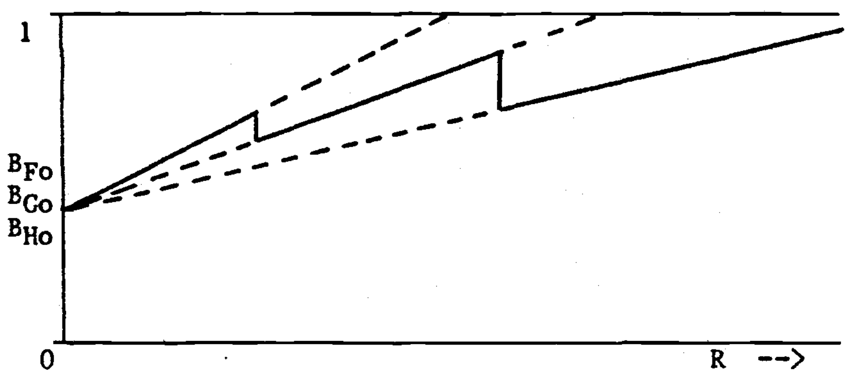

Fig. 3.3

Labor Share of Output

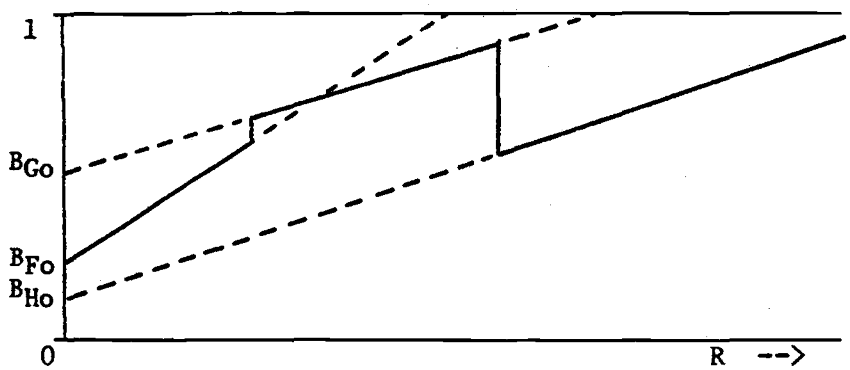

Fig. 3.1: Profit per acre as a function of distance, R, from center, for fruit, grain, and horses.

Fig. 3.2: Labor share of output as function of distance, R, assuming $B_{Fo} = B_{Go} = B_{Ho}$, -- the sawtooth curve.

Fig. 3.3: Labor share of output as function of distance, R, assuming $B_{Go} \gg B_{Fo} > B_{Ho}$.

B_{Go}, B_G -- labor share for grain, without and with transportation.

B_{Ho}, B_H -- labor share for horses, without and with transportation.

There is no necessary relationship between the natural intrinsic labor intensity of fruit, grain, and horses, that is, between their labor shares at w, B_{Fo}, B_{Go}, and B_{Ho}, not counting transportation. However, labor share including transportation rises linearly with R for all three crops.

As obvious from (4.1) - (4.3), profit per acre is just output per acre times one minus labor share. For example for fruit:

(4.4) $$\frac{P_F}{T} = \frac{f}{T}[1 - B_F]$$

So for each crop, profit per acre falls linearly as labor share rises linearly with R.

Figures 3.2 and 3.3 show labor shares for fruit, grain and horses--rising from their minimum values at the center, B_{Fo}, B_{Go}, and B_{Ho}; to their maximum value of 1, where profit becomes zero. Fig. 3.2 assumes for simplicity that natural intrinsic labor intensity is the same for all three crops, so $B_{Fo} = B_{Go} = B_{Ho}$. Fig. 3.3 assumes that fruit has a much lower natural intrinsic labor intensity than grain, so $B_{Go} >> B_{Fo} > B_{Ho}$.

In both figures, labor share of output for each crop appears as a solid line in the region that the crop is produced, and a dashed line in the regions the crop isn't produced. So the solid line in both figures shows the labor share of output, and hence the intrinsic labor intensity of production, as a function of distance R from the center.

In both figures, labor share rises in the region of each crop. In Fig. 3.2, it drops again at the boundary between fruit and grain, and between grain and horses. So labor share, and hence intrinsic labor intensity, moves outward in a sawtooth. Fig. 3.3 shows that if the natural intrinsic labor intensity of fruit is assumed very much lower than that of grain, labor share jogs upward at the fruit-grain boundary.

There is no a priori reason to assume that more centrally located activities on the average show a lower natural intrinsic labor intensity than more peripheral ones. So in practice it should happen only occasionally that the inner one of two activities has natural intrinsic labor intensity so much lower that intrinsic labor intensity including transportation jumps up instead of dropping at the boundary between activities. In general, the sawtooth pattern should prevail.

Locational Advantage as a Function of Wealth:

The precise location of the teeth of the sawtooth pattern depends on the assumed market wage. For as a moment's reflections shows, the higher the wage, the smaller the bullseye, that is, the closer to market the boundaries between activities. And the higher the wage, the higher and more steeply rising the labor share of output for any activity.

Now, what happens in a world with transactions costs, where richer farmers have a higher wage than poorer ones?

Comparative advantage says that richer farmers locate on land suited to the lowest intrinsic labor intensity production: the fruit land, grain land, and horse land closest to market. However, the precise location of the fruit-grain and grain-horse boundaries becomes unclear. For the same land that poorer farmers consider good for fruit looks good for grain to richer farmers; and the same land that poorer farmers consider good for grain looks good for horses to richer farmers. In short, between fruit and grain, and between grain and horses lie regions where highest and best use depends on the wealth of the owner. The actual boundaries must fall in these regions, if indeed there are distinct boundaries.

B. Wealth, Transportation Cost and Output per Acre:

It's an unrealistic but harmless convention of classic location theory that location does not affect output per acre. Consequently, as assumed in A., transportation cost per acre is fixed. And fixed transportation costs make for neat, straight line graphics.

But if transportation labor does vary with output, then within any activity, output per acre falls with distance from the center. This is pretty obvious, as transportation lowers the net value of produce delivered to market. But to show this formally, consider just fruit, dropping subscripts. Transportation cost is now proportional to output, f(T,A), as well as wage, w, distance, R, and cost coefficient, c.

A farmer maximizes profits:

$$(4.5) \qquad P = f(T,A) - w[A + cRf(T,A)] = f(T,A)(1-wcR) - wA$$

obtaining the first-order condition:

$$(4.6) \qquad f_2(1 - wcR) - w = 0$$

So marginal product of applied labor now exceeds the wage, except at the central place where $R = 0$.

Consequently, output per acre must fall as distance from the center increases. That is:

$$(4.7) \qquad \frac{df}{dR} = \frac{f_2{}^2 wc}{f_{22}} < 0$$

And instead of a straight line, profit now falls in a curve convex to the origin:

$$(4.8) \qquad \frac{d}{dR}\left(\frac{P}{T}\right) = -\frac{wc}{T}[f - wcR] < 0$$

Comparative Advantage and Output per Acre:

So in a perfect market, transportation costs make output per acre fall with distance from the center.

In a market with transactions costs, as shown in Chp. 1, richer farmers get lower output per acre than poorer ones from the same quality land.

But now comparative advantage says that richer farmers occupy better located land, which in a perfect market would yield higher output per acre.

So in net, it's not possible to predict whether richer farmers on better land get higher or lower output per acre than poorer farmers on worse land.

3.5 Wealth and Supervision Costs

It seems intuitively obvious that production requiring a higher rate of supervision, all else being equal, has higher intrinsic labor intensity--at least for the owner or manager of a firm. That is, the more supervision required, the higher the owner or manager's labor share of output net of other costs, holding constant his marginal product of labor. In fact, this necessarily happens only given the very plausible assumption in Chp. 1, Sec. 3, 5e, that as more labor crowds onto a given piece of land, labor share of output falls. That is, if production depends on land and (hired) labor, f(T,H), then:

$$(5.1) \qquad \frac{d}{dH}\left(\frac{f_2 H}{f}\right) < 0$$

That a higher rate of supervision raises the owner or manager's labor share of output can be shown using the large landlord model of Sec. 1.8. Recall that the large landlord supervises his employees at rate k, so that his own labor $L = kH$. From the KT conditions of Sec. 1.8, $0 \leq k \leq 1 - v/w$, where v is the hired labor wage, and w the landlord's wage, $w > v$.

The landlord's firm maximizes profit:

$$(5.2) \qquad P = f(T,H) - vH - wL$$
$$= f(T,H) - (v + kw)H$$

obtaining the first-order condition:

$$(5.3) \qquad f_2 - v - kw = 0$$

To find out what happens to the landlord's labor share of net output as k increases, we must hold constant the landlord's marginal product of labor, $(f_2 - v)/k = w$.

If k increases, holding w constant, H falls:

$$(5.4) \qquad \frac{dH}{dk} = \frac{w}{f_{22}} < 0$$

Therefore, from (5.1) total labor share of output must rise:

$$(5.5) \qquad \frac{d}{dk}\left(\frac{vH + wL}{f}\right) = \frac{d}{dk}\left(\frac{f_2H}{f}\right) = \frac{d}{dH}\left(\frac{f_2H}{f}\right)\frac{dH}{dk} > 0$$

Now what about the landlord's labor share of output net of payments to employees?:

$$(5.6) \qquad \frac{d}{dk}\left(\frac{wL}{f - vH}\right) = w\frac{d}{dk}\frac{kH}{f - vH} = \frac{wH}{f - vH}\left[1 + \frac{(f - f_2H)}{f- vH}\frac{k}{H}\frac{dH}{dk}\right]$$

This is obviously > 0 for k = 0 or very small. What happens at the other end of the range, as $k \rightarrow 1 - v/w$, and $f_2 \rightarrow w$? From the assumption that labor share falls as H increases, it can easily be shown that:

$$(5.7) \qquad \frac{dH}{dk} = \frac{w}{f_{22}} > -\frac{wH}{f_2}\left(\frac{f}{f - f_2H}\right) = -\frac{fH}{f - f_2H} \quad \text{for } f_2 = w$$

And from (5.7) it follows that at $f_2 = w$:

$$(5.8) \qquad \frac{d}{dk}\left(\frac{wL}{f - vH}\right) > \frac{wH[f(1-k)-vH]}{(f-vH)^2} \geq 0 \quad \text{as long as } P \geq 0$$

So presumably the landlord's labor share of net output increases everywhere as k increases, at a given wage w for the landlord. So the higher k, the higher the intrinsic labor intensity of production.

It therefore follows from Sec. 3.3 that a richer landlord has a comparative advantage in production that requires a lower rate of supervision.

3.6 Appendix 1: Marginal Product of Land and Comparative Advantage[D]

Assumption:

Suppose there are two landowners, X and Y, and two kinds of land, F-land and G-land. The two landowners initially own F-land and G-land in the same proportions.

Definition:

Suppose X and Y can trade F for G land, such that after trade X has a larger proportion of F-land, and Y has a larger proportion of G-land, and both are better off. Then X has a comparative advantage in F-land, and Y has a comparative advantage in G-land.

Proposition:

X has a comparative advantage in F-land, and Y has a comparative advantage in G-land if and only if X's ratio of marginal products of F to G land exceeds Y's ratio of marginal products of F to G land.

To Prove:

The proposition follows directly from proving that:

If the two landowners X and Y each own some F-land and some G-land, not necessarily in the same proportions, they can trade to mutual advantage if and only if they have different ratios of marginal product of F-land to marginal product of G-land. X will receive F-land and give G-land if and only if his ratio exceeds Y's ratio.

Notation:

X -- superscript for landowner X

Y -- superscript for landowner Y

F^X, F^Y -- F-land belonging to X and Y

G^X, G^Y -- G-land belonging to X and Y

$L_F{}^X$, $L_G{}^Y$ -- X and Y's labor on F-land

$L_G{}^X$, $L_G{}^Y$ -- X and Y's labor on G-land

$f(F^X, L_F{}^X)$, $f(F^Y, L_F{}^Y) = f^X$, f^Y -- X and Y's output from F-land

$g(G^X, L_G{}^X)$, $g(G^Y, L_G{}^Y) = g^X$, g^Y -- X and Y's output from G-land

$f^X + g^X = Q^X$ -- X's output and consumption

$f^Y + g^Y = Q^Y$ -- Y's output and consumption

$f_1{}^X$, $f_1{}^Y$ -- Y and Y's marginal products of F-land

$g_1{}^X$, $g_1{}^Y$ -- X and Y's marginal products of G-land

$r^X = f_1{}^X / g_1{}^X$ -- X's ratio of marginal products of land

$r^Y = f_1{}^Y / g_1{}^Y$ -- Y's ratio of marginal products of land

$R = dG/dF$ -- proportion in which X and Y trade increments of F and G land

$R^* = dG^*/dF^*$ -- proportion which leaves both X and Y better off

<u>Proof</u>

Suppose that:

(6.1) $$r^X > r^Y$$

Then there exists a proportion $R^* = dG^*/dF^*$ at which X and Y can trade increments of land dF^* and dG^* and leave both better off. X will accept F-land and give G-land.

Say that X gets an arbitrary dF and gives Y an arbitrary dG in exchange. If neither alters the labor he applies to his land, then their change in consumption is:

(6.2) $$dQ^X = f_1{}^X dF - g_1{}^X dG = (r^X - R) g_1{}^X dF$$

6.3) $$dQ^Y = -f_l^Y dF + g_l^Y dG = (-r^Y + R)g_l^Y dF$$

Both X and Y are better off if their consumption increases (or does not decrease) while their labor does not change. So for both to be better off:

(6.4) $$dO^X > 0 \Rightarrow r^X > R$$

(6.5) $$dQ^Y > 0 \Rightarrow R > r^Y$$

So a proportion R* that leaves both X and Y better off (in fact infinitely many proportions) exists if and only if $r^X > r^Y$, so that

(6.6) $$r^X > R^* > r^Y$$

OED

3.7 Appendix 2: Properties of Functions in Two Variables; Homogeneous Functions; Linear Homogeneous Functions[D]

Definitions:

$f(T,L)$ = a function in two variables, such that f_1, f_2, f_{12} > 0, and f_{11}, $f_{22} < 0$.

$$B = \frac{f_2 L}{f} = \text{labor share of output}$$

$$N = \frac{f_1 T}{f} = \text{land share of output}$$

$$e = N + B = \text{total elasticity of output}$$

Output as Function of Land Size, T, and Wage, w:

Suppose that the marginal product of labor equals a given wage, w:

(7.1) $$w - f_2 = 0$$

Then output depends on land size, T, and wage, w:

(7.2) $$\left.\frac{dL}{dT}\right|_w = -\frac{f_{12}}{f_{22}} > 0$$

(7.3) $$\left.\frac{dL}{dw}\right|_T = \frac{1}{f_{22}} < 0$$

So:

(7.4) $$\left.\frac{df}{dT}\right|_w = f_1 - w\frac{f_{12}}{f_{22}} > 0$$

(7.5) $$\left.\frac{df}{dw}\right|_T = \frac{w}{f_{22}} < 0$$

Total Elasticity of Output (Scale) as Function of Land, T, and Wage, w:

Total elasticity of output, e, equals the sum of the "shares" of output -- which equals one only for constant returns to scale:

$$(7.6)\qquad e = \frac{f_1 T}{f} + \frac{f_2 L}{f} = N + B \quad \begin{cases} = 1 & \text{constant returns} \\ > 1 & \text{increasing returns} \\ < 1 & \text{decreasing returns} \end{cases}$$

Total elasticity implicitly depends on land size, T, and wage, $w = f_2$:

$$(7.7)\qquad \left.\frac{de}{dT}\right|_w = \frac{1}{f}\left[\, T\left[f_{11} - \frac{(f_{12})^2}{f_{22}}\right] + (1-e)\left[f_1 - w\,\frac{f_{12}}{f_{22}}\right]\,\right]$$

≤ 0 for most reasonable functions. That is, returns to scale diminish as size increases, -- though with w held constant, L increases at the same rate as T only for linear homogeneous functions.

An expression useful in Sec. 3.3 derives from (7.7):

$$(7.8)\qquad \frac{f_{11} - \frac{(f_{12})^2}{f_{22}}}{f_1} = \frac{1}{e - B}\left.\frac{de}{dT}\right|_w + \frac{e-1}{T}\left(1 - \frac{w}{f_1}\,\frac{f_{12}}{f_{22}}\right)$$

$$(7.9)\qquad = \frac{1}{e - B}\left[\left.\frac{de}{dT}\right|_w + \frac{e-1}{f}\left.\frac{df}{dT}\right|_w\right]$$

$$(7.10)\qquad = \frac{1}{e - B}\left.\frac{de}{dT}\right|_w + \frac{e - 1}{(e-B)(1-B)}\left.\frac{dB}{dT}\right|_w + \frac{e-1}{1-B}\,\frac{1}{T}$$

For total elasticity as a function of wage, $w = f_2$:

$$(7.11)\qquad \left.\frac{de}{dw}\right|_T = \frac{1}{f}\left[\,\frac{f_{12}}{f_{22}}\,T + L + \frac{w(1-e)}{f_{22}}\,\right]$$

≥ 0, if scale falls as size increases, because an increase in w reduces L.

Another expression useful in Sec. 3.3 derives from (7.11):

$$(7.12)\qquad \frac{1}{f_1}\,\frac{f_{12}}{f_{22}} = \frac{B}{e - B}\left[\left.\frac{de}{dw}\right|_T - \frac{1}{w}\left[1 + \frac{w(1-e)}{f_{22} L}\right]\,\right]$$

$$(7.13) \qquad = \frac{B}{e-B}\left[\left.\frac{de}{dw}\right|_T - \frac{1}{w}\right] + \frac{e-1}{e-B}\frac{1}{f}\left.\frac{df}{dw}\right|_T$$

$$(7.14) \qquad = \frac{1}{e-B}\left.\frac{de}{dw}\right|_T + \frac{e-1}{(e-B)(1-B)}\left.\frac{dB}{dw}\right|_T - \frac{B}{1-B}\frac{1}{w}$$

<u>Homogeneous Functions:</u>

Suppose the function of two variables, $f(T,L)$, is homogeneous of degree e, where e is the (constant) total elasticity of output. Then, if t is a constant multiplying the two variables:

$$(7.15) \qquad f(tT,tL) = t^e f(T,L)$$

Differentiation of (7.15) with respect to T and L yields:

$$(7.16) \qquad f_1(tT,tL) = t^{e-1} f_1(T,L)$$

$$(7.17) \qquad f_2(tT,tL) = t^{e-1} f_1(T,L)$$

The first derivatives are homogeneous of degree e-1.

From (7.6), (or from differentiating (7.15) wrt. t):

$$(7.18) \qquad f_1T + f_2L = ef(T,L)$$

Total differentiation of (7.18) yields:

$$(7.19) \qquad (f_1+f_{11}T+f_{12}L)dT + (f_2+f_{12}T+f_{22}L)\,dL = e(f_1dT + f_2dL)$$

From which it follows that:

$$(7.20) \qquad f_{11}T + f_{12}L = (e-1)f_1$$

$$(7.21) \qquad f_{12}T + f_{22}L = (e-1)f_2 \qquad \text{or,} \quad -\frac{f_{12}}{f_{22}} = \frac{L}{T} + \frac{(1-e)f_2}{f_{22}T}$$

From these two expressions, or from (7.8) - (7.10) with $\left.\frac{de}{dT}\right|_w = 0$ the expressions useful in Sec. 3.3 easily derive:

(7.22) $$\frac{f_{11} - \frac{(f_{12})^2}{f_{22}}}{f_1} = \frac{1-e}{T}\left[1 - \frac{f_{12}w}{f_{22}f_1}\right] \quad \begin{matrix} < 0 & e > 1 \\ = 0 & e = 1 \\ > 0 & e < 1 \end{matrix}$$

(7.23) $$= \frac{1}{e - B}\,\frac{e-1}{f}\left.\frac{df}{dT}\right|_w = \frac{e-1}{(e-B)}\,\frac{1}{T}\left[e + \frac{(1-e)wB}{(e-B)f_{22}L}\right]$$

(7.24) $$\underset{approx}{=} \frac{e(e-1)}{T(e-B)} \qquad \text{for e close to 1, and/or small w.}$$

From (7.20) and (7.21), or from (7.12) - (7.14) with $\left.\frac{de}{dw}\right|_T = 0,$ it also follows that:

(7.25) $$\frac{1}{f_1}\,\frac{f_{12}}{f_{22}} = -\frac{B}{e - B}\,\frac{1}{w}\left[1 + \frac{w(1-e)}{f_{22}L}\right]$$

(7.26) $$= -\frac{B}{e - B}\,\frac{1}{w} + \frac{e-1}{e-B}\,\frac{1}{f}\left.\frac{df}{dw}\right|_T$$

(7.27) $$\underset{approx}{=} -\frac{B}{e - B}\,\frac{1}{w} \qquad \text{for e close to 1, and/or small w}$$

Linear Homogeneous Functions:

For a linear homogeneous function, e = 1. Therefore, from (7.20) and (7.21):

(7.28) $$\frac{f_{11}}{f_{12}} = \frac{f_{12}}{f_{22}} = \frac{L}{T}$$

(7.22) and (7.27) then become:

(7.29) $$\frac{f_{11} - \frac{(f_{12})^2}{f_{22}}}{f_1} = 0$$

(7.30) $$\frac{1}{f_1}\,\frac{f_{12}}{f_{22}} = -\frac{B}{1 - B}\,\frac{1}{w}$$