CHAPTER 2

DISTRIBUTION OF FIRM SIZE, ECONOMIC CLASSES, AND OTHER CONSEQUENCES OF INEQUALITY, WITH AND WITHOUT TRANSACTIONS COSTS

The Clones, like dutiful laboratory animals, lend themselves to a variety of little experiments that illuminate further the effects of wealth and transactions costs, and the absurdity of a world without transactions costs. The self-sufficient farmers and the large landlords serve as guinea pigs with transactions costs, while the peasants serve the same function without transactions costs. Sec. 2.1 summarizes these experiments. Sec. 2.2 draws some broader implications for behavior of the firm, firm size, and social and economic class.

2.1 Summary[A]

Natural Ability (Sec. 2.3):

Suppose (in temporary violation of their basic character) we vary the natural ability of Clones--so that the actual labor delivered by an individual equals the hours he works times an exogenous ability factor, b. Then, with or without transactions costs, greater ability raises a Clone's effective labor supply (hours times ability factor). But with transactions costs, a more able Clone applies more effective labor to a given piece of land, getting a higher output per acre. Without transactions costs, the amount of labor applied, and output per acre remain independent of the ability of the owner! Given transactions costs, a more able rich Clone works longer hours than a less able rich Clone, but a more able poor Clone works fewer hours than a less able poor Clone. Without transactions costs, a more able Clone always works longer than a less able Clone of the same wealth.

Education (Sec. 2.4):

Suppose a Clone farmer can extent his personal labor supply by selling an amount of his land, E, for education, which multiplies his hours of labor by an amount e(E), subject to diminishing returns. Then, with transactions costs, a richer Clone always gets more education. A larger firm always buys more employee training. This makes perfect sense; education allows richer individuals and bigger firms to trade what they have relatively (and absolutely) more of--land--for what they have relatively less of--labor. Without transactions costs, a richer individual always gets less education!--because he works less. Therefore, a richer individual actually earns a lower wage than a poorer one! As for firms, without transactions costs (and necessarily assuming linear homogeneous production), firm size does not affect employee training.

Supervision Rate and Performance (Sec. 2.5):

The landlords of Chp. 1 faced a fixed, exogenous rate of supervision. Suppose now that a large landlord can choose his rate of supervision. The more he supervises, the better his employees perform, that is, the greater their effective labor supply. (Presumably they work faster and more reliably). Given this assumption, the richer the landlord, the less he supervises, and the worse his employees perform. (However, the effective supply of hired labor still increases with wealth). Obviously, absent transactions costs, an owner's supervision does not affect employee performance!

Skill, Performance, and Rate of Pay (Sec. 2.6):

Suppose a large Clone landlord can improve his employees' performance by paying better. The higher the pay he offers, subject to diminishing returns, the more skilled the employees he gets, and the better employees of given skill perform. With this assumption, then the richer the landlord or larger the firm, the higher the pay of employees, and the better their performance. But absent transactions costs, wealth or firm size does not affect rate of pay or level of employee skill.

So there are opposing pressures on the performance of employees as wealth or firm size increase. In combination, a richer landlord supervises less but pays more--not clearly getting better or worse performance.

Rental and Leverage, With and Without Transactions Costs:

In the real world, the quantity of assets people or firms rent or borrow rises with wealth and firm size, though not as fast. So the richer the person or larger the firm, the lower the leverage: the ratio of rented or borrowed assets to owned assets. Moreover, rental and interest rates fall as wealth or firm size increase--well-known symptoms of capital market failure.

To reproduce this familiar pattern in Cloneland--a rise in debt with equity, but fall in ratio of debt to equity, and in rental or interest rates--we must assume transactions costs proportional to debt to equity ratio. This is quite a reasonable assumption if the transactions costs to lenders, either in supervising a loan or in insuring against loss, rise with the riskiness of the loan. This riskiness presumably rises with debt to equity ratio of the borrower.

With no transactions costs, and therefore assuming linear homogeneous production, there must necessarily be one fixed "market" rate of rental equal to the marginal product of land, just as there is a fixed wage rate, equal to the marginal product of labor. Under these circumstances assume for a moment the peasant (paradigm of a transactions cost-less world) rents additional land instead of working for hire on another's land. Then, since a richer peasant works less than a poorer one, he operates a smaller farm than a richer one! So instead of rising with wealth, debt falls with wealth, and debt to equity plummets, so fast that farm size falls. (Of course how much renting versus hiring the peasant does is indeterminate absent all transactions costs, as then the peasant has no necessary connection with the operation of his farm).

But if, due implicitly to transactions costs, the rental rate rises substantially with the ratio of rented to owned land--that produces the right results for a farmer or landlord permitted to rent additional land. That is, rented land rises with owned land, but ratio of rented to owned land falls, as does rental rate. In addition, under this assumption, the marginal product of rented land exceeds the rental rate--just as the marginal product of labor exceeds the wage paid employees.

Parcel Size, Supervision Rate and Reliability of Lessees (Sec. 2.8):

In the real world, larger landlords lease out larger parcels, bigger banks make bigger loans, and larger investors own larger blocks of particular stocks and bonds. Larger entities are more diversified than smaller ones, but due to this propensity for larger parcels etc., diversification does not rise as fast as wealth and firm size.

Larger landlords and banks also prefer "better quality" clients, to whom they charge lower rent or interest. (Bigger investors prefer blue chip stocks, whose higher price to earnings ratios, inverted, mean lower earnings per dollar invested. However the comparison of different size market investors is complicated by large economies of scale in access. See Chp. 14 for discussion.)

To reproduce this pattern in Cloneland--rental rate falling and parcel size rising with wealth of landlord, so that diversification does not increase as fast as wealth--requires assuming transactions costs.

So suppose a landlord can rent out his land in an arbitrary number of parcels, but each tenant requires a certain amount of supervision. If the rent landlords could get didn't vary with size of parcel, there would be no advantage to breaking up land into several parcels rented to different tenants. But suppose that market rent falls as parcel size increases--because larger parcels go to larger, less-leveraged tenants. Then the more land a landlord owns, the more parcels he rents out, but the number of parcels does not rise as fast as wealth. So parcel size increases with wealth, and rent obtained per acre falls.

Three variations on this model, too obvious to construct, yield additional predictions:

Suppose landowners vary in ability. Then a more able landowner rents out smaller parcels and obtains higher rent for the same total area of land.

Now suppose a landowner can vary his supervision per parcel. But the less he supervises the less rent he can expect to collect. That is, expected rent falls due to more defaults, and the variance of rent rises. Then the more land a landowner has, the less he supervises, the less rent he collects, and more variance he must tolerate.

But now suppose the landowner can choose the "quality" of his lessees. However, more reliable lessees demand a lower rent. Then the more land a landowner has, the more reliable his lessees, and the lower the rent he collects.

So a larger landowner will lease larger parcels at lower rents to more reliable lessees, but supervise them less. The expected collection rate, and the reliability of lessees, may rise or fall with land size--depending on which factor dominates.

Firm Size and Natural Ability (Sec. 2.9):

Suppose we vary the ability of a large landlord, who can both hire employees and rent additional land, subject to transactions costs. Then the more able a landlord of given wealth, the more employees he hires and the more additional land he rents. However, the ratio of labor to land rises and hence output per acre and leverage rise with ability, while output per manhour falls.

So given market wage and rental levels, then three factors fully determine the size of a landlord's farm as measured by area of land operated, number of employees, or output: 1) the area of land owned, 2) the landlord's natural ability, and 3) the underlying production function--the greater the economies of scale or smaller the diseconomies, the larger the farm.

2.2 Determination of Firm Size and Economic Class[A]

Determination of Firm Size (Conceptual vs. Operating Firms):

Every person in the Clone economy owns a conceptual firm. This firm directly owns his land (for convenience in modelling). And it can hire his labor free of transactions costs. The conceptual firm may hire out its owner's labor and/or rent out his land, or hire additional labor and/or rent additional land. The size of a person's conceptual firm, measured by land area, output, profit, or labor supply, depends on his wealth in land area owned, and his natural ability.

An operating firm, on the other hand, consists of a piece of land operated as a unit, together with direct labor and supervisory labor (if any): a "farm". It may or may not coincide with a conceptual firm. For example, when the peasant both works on his own land and for hire elsewhere, his conceptual firm includes or owns a smaller firm that operates his land. A landless peasant, or a landowner who rents out all his land to other firms do not own operating firms at all.

Absent transactions costs in either hiring labor or renting land, operating firm size depends neither on owners' wealth or ability, but solely on scale in the underlying production function. As described in Chp. 1, diminishing returns would splinter the economy into a zillion firmlets, while increasing returns would congeal it into one great corporate blob. But both diminishing and increasing returns leave the product not adding up to factor payments. A linear homogeneous production function leaves firm size indeterminate, while if the production function shows scale economies at small sizes and diseconomies at large sizes, the economy splits into identical firms all of the size giving constant returns to scale.

However, with transactions costs, wealth and natural ability affect operating firm size. The greater the transactions costs, the more closely operating and conceptual firms must coincide. In the limit where transactions costs prevent all hiring and renting, operating and conceptual firms become identical. Then the distribution of wealth and natural ability completely determine the distribution of operating firm size.

What happens at an intermediate level of transactions costs? Presumably, the greater the economies of scale in the underlying production function, the fewer the number of operating firms, and the more unequal their size distribution. (Economies of scale would raise the wages and rents offered by big firms, pulling more labor and land away from small ones). Underlying diseconomies should increase the number of operating firms, and make sizes more equal.

So, in a world with transactions costs, size of operating firms depends positively on wealth of the owner(s), ability of the owner(s) or managers, and economies of scale in production technology. A lack of transactions costs would rule out this commonplace relationship.

Economic Classes:

As described, the results of this chapter suggest that transactions costs cause a sort of economic stratification. Education rises with wealth. And wealthier persons deal preferrentially with one another.

The results also suggest an economic rationale for nepotism, "old boys' networks", and class discrimination in hiring or renting: employers or lessors may find relatives or persons of similar background more reliable--and can save supervisory labor (at a cost in gross or net output) by preferring such persons.

2.3 Natural Ability, With and Without Transactions Costs[C]

Suppose natural ability, "b", varies from one individual to another. Natural ability multiplies an individual's labor supply. So if his actual labor supply is "L", his effective labor supply is "bL".

Then, with transactions costs, a more able person produces more from a given size piece of land; without transactions costs, production does not depend on ability. These and other contrasts are developed in subsections (A) and (B) below.

(A) Natural Ability With Transactions Costs (from 1.5):

Suppose the self-sufficient farmers of Sec. 1.5 vary in an exogenous ability factor, b. Then their firms must maximize profit as a function of effective labor supply, bL:

$$(3.1) \quad P = f(T,bL) - wL$$

obtaining the first order condition:

$$(3.2) \quad w - bf_2 = 0$$

which gives the firm's demand for labor. This can be combined with the landowner's labor supply equation (derived from Sec. 1.4, (4.2) and (4.3)):

$$(3.3) \quad L = a(P+wD,w)$$

The combination yields labor supplied, output, and other variables as functions of the original exogenous variable, land size T, and a new exogenous variable, ability, b. Partial derivatives with respect to ability show the effect of greater ability, holding land size constant:

Partial Derivatives with Respect to Natural Ability, b:

$$(3.4)\qquad J = 1 - [a_1 Z + a_2] b^2 f_{22} > 0 \qquad \text{(determinant)}$$

$$(3.5)\qquad \frac{d}{db}(bL) = \frac{(a_1 D + a_2) w}{J} > 0 \qquad \text{effective labor supply increases}$$

$$(3.6)\qquad \frac{d}{db} f(T,bL) = f_2 \frac{d}{db}(bL) > 0 \qquad \text{output increases}$$

$$(3.7)\qquad \frac{d}{db} f_2(T,bL) = f_{22} \frac{d}{db}(bL) < 0 \qquad \text{MP labor decreases}$$

$$(3.8)\qquad \frac{dL}{db} = \frac{(a_1 D + a_2)(f_2 + bLf_{22}) - a_1 bL^2 f_{22}}{J} \qquad > 0 \text{ then } < 0$$

$$(3.9)\qquad \frac{dw}{db} = \frac{(f_2 + bLf_{22}) + b^2 f_{22} a_1 L f_2}{J} \qquad > 0 \text{ then } < 0$$

Both dL/db and dw/db contain the expression: $f_2 + bLf_{22}$. This expression is > 0 for a low ratio of effective labor to land, so f_2 is high. It becomes < 0 for a high ratio of effective labor to land. So as natural ability increases, the landowner's labor supply and wage first increase, and then decrease. The labor supply begins to decrease while the wage still rises.

(B) Natural Ability Without Transactions Costs (from Sec. 1.6):

Suppose the peasants of Sec. 1.6 vary in ability, b, and that the exogenous market wage fully reflects this variation, so by working H hours for hire, they earn vbH. Then their firms maximize profit as a function of effective labor supply bL = bS + bH, where S is labor the peasant does himself on his own land:

$$(3.10)\qquad P = f(T,bS) + vbH - w(S + H)$$

obtaining first order conditions:

$$\left.\begin{array}{ll}(3.11) & w - bf_2 = 0 \\ (3.12) & w - bv = 0\end{array}\right\} \quad (3.13) \quad f_2 = v = \text{constant}$$

The equilibrium conditions immediately show that the marginal product of labor remains fixed at the given market wage, v. The peasant's wage then depends only on his ability, b.

The firm's equilibrium conditions and the peasant's labor supply equation (3.3) together implicitly give output and other variables as functions of three exogenous variables, two old ones, T and v, and the new one, natural ability, b. Derivatives show the effect of increased ability, holding constant land size T and market wage v:

<u>Partial Derivatives with Respect to Ability, b:</u>

(3.15)	$\frac{d}{db} bS = 0$	effective labor on land remains constant
(3.16)	$\frac{d}{db} f(T,bS) = 0$	output remains constant
(3.17)	$\frac{d}{db} f_2(T,bS) = 0$	MP labor remains constant
(3.18)	$\frac{dS}{db} = -\frac{L}{b} < 0$	labor on land falls
(3.19)	$\frac{dH}{db} = \frac{L}{b} + v(a_1D+a_2) > 0$	hired labor rises
(3.20)	$\frac{dL}{db} = v(a_1D+a_2) > 0$	total labor rises
(3.21)	$\frac{dw}{db} = v > 0$	wage rises

2.4 Education, With and Without Transactions Costs[c]

Suppose that, before production occurs, a landowner can trade off a portion of his land, E, in exchange for an increase in his education, e(E). Education multiplies his labor supply, so that if his actual labor supply is L, his effective supply is eL. But this investment in education shows diminishing returns, so that $e > 0$, $e' > 0$, but $e'' < 0$. Production then appears as a function of land, labor, and education: $f(T-E,e(E)L)$. Only land is exogenous.

Then with transactions costs, the richer a landowner, the more education he gets. Without transactions costs, the richer a landowner, the less education he gets and hence the lower his wage! These and other contrasts appear in subsections (A) and (B).

(A) Education With Transactions Costs (from 1.5):

If the self-sufficient farmer of Sec. 1.5 can trade some land for education, then his firm maximizes profit:

$$P = f(T-E,eL) - wL \tag{4.1}$$

obtaining the first-order conditions:

$$w - ef_2 = 0 \tag{4.2}$$

$$f_2e'L - f_1 = 0 \tag{4.3}$$

(4.3) says that the marginal product of land used in production equals the marginal product of land exchanged for education.

The first-order conditions and the landowner's labor supply equation (3.3) ($L = a(P+wD,w)$) together implicitly give education and other variables as a function of land size. Derivatives show the effect of an increase in land size:

Partial Derivatives with Respect to T, Land Size:

(4.4) $$\frac{dE}{dT} = \frac{1}{JJ_1}[\ (a_1Z+a_2)[ee'f_2f_{12} + e^2(f_{11}f_{22}-(f_{12})^2)] - f_{11} + f_{12}e'L - a_1f_1J_2\ ] \quad > 0 \quad *, **$$

(4.5) $$\frac{dL}{dT} = \frac{1}{J}[\ a_1f_1 + (a_1Z+a_2)[ef_{12} - (f_{12}e'L-f_{11})J_2/J_1]\ ] \quad *$$

> 0 by assumption of no bkwd bending

(4.6) $$\frac{dw}{dT} = \frac{1}{JJ_1}[\ -ee'L[f_{11}f_{22}-(f_{12})^2] - f_2(ef_{12}e''L+e'f_{11}-e'^2Lf_{12}) + a_1f_1J_1(e^2f_{22} + J_2^2/J_1)\ ] \quad > 0 \quad *, **$$

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*Define:

$$J_1 = 2f_{12}e'L - f_{22}e'^2L^2 - f_2e''L - f_{11} \qquad > 0$$

$$J_2 = ef_{12} - e'(f_2+eLf_{22}) \qquad > 0$$

(> 0 for small f_2, large f_{22}, where eL/(T-E) is large
> 0 for large e, e' -> 0, where eL/(T-E) is small.)

Then:

$$J = 1 - [a_1Z+a_2](e^2f_{22} + J_2^2/J_1) \quad > 0$$

since

$$e^2f_{22} + J_2^2/J_1 = \frac{1}{J_1}[-e^2(f_{11}f_{22}-(f_{12})^2) + f_2(e'^2f_2 + 2e'^2eLf_{22} - 2ee'f_{12} - e^2e''Lf_{22})\ ] \quad < 0$$

The first term depends on scale and is ignored as relatively unimportant. Else, only $e'^2f_2 > 0$, but this = 0 for very large or very small labor to land ratios, so ignore it.

** Assuming relative insignificance of $f_{11}f_{22} - (f_{12})^2$. Ie., there are insufficient economies of scale to outweigh advantages of more education.

So expenditure for education, wage and labor supply rise with land size. A simple transformation shows that the other results of Section 1.5 must continue to hold too: If $g(T,L,E) \equiv f(T-E,eL)$, then g_1, g_2 and $g_{12} > 0$, and g_{11} and $g_{22} < 0$. So what applies to an arbitrary function, f, meeting these conditions, also applies to g.

Employee Education:

Consider the large landlord model of Sec. 1.8, modified by the possibility of trading some land for employee education. The firm maximizes profit:

$$P = f(T-E,eH) - (v + kw)H$$

Defining $h(T,L,E) \equiv f(T-E,eH) - vH = f(T-E,eL/k) - vL/k$, then h_1, h_2, and $h_{12} > 0$, and h_{11}, and $h_{22} < 0$. So everything holds for employee education that holds for the owner's education.

The same also holds assuming that employee education increases employees' wage--because employee education is sufficiently unspecialized to be valuable to other potential employers, as can be shown by defining:

$$j(T,L,E) \equiv f(T-E,eH) - veH = f(T-E,eL/k) - veL/k.$$

So bigger firms provide more employee education and training.

(B) Education Without Transactions Costs (from Sec. 1.6):

The peasant's firm maximizes profit, where the peasant's labor supply $L = S + H$, labor the peasant applies to his own land, plus labor hired out:

$$(4.8) \qquad P = f(T-E, eS) + veH - w(S + H)$$

obtaining first-order conditions:

$$\left.\begin{array}{ll} (4.9) & w - ef_2 = 0 \\ & \\ (4.10) & w - ev = 0 \end{array}\right\rangle \quad (4.12) \quad f_2 = v = \text{constant}$$

$$(4.11) \qquad e'vL - f_1 = 0$$

So the marginal product of labor, f_2, remains fixed at the market wage, v. Assuming, necessarily, a linear homogeneous production function, the marginal product of land, f_1 is also fixed, and hence so is $e'vL$, the marginal product of land traded for education.

The first-order conditions and the peasant's labor supply equation 3.3 ($L = a(P+wD,w)$) together implicitly give education and other variables as a function of land size. Derivatives show the effect of an increase in land size:

Partial Derivatives with Respect to T, Land Size:

$$(4.13) \qquad \frac{dE}{dT} = -a_1 f_1 \frac{e'}{e''L + (a_1 Z + a_2)e'^2 v} < 0 \; *$$

$$(4.14) \qquad \frac{dL}{dT} = a_1 f_1 \frac{e''L}{e''L + (a_1 Z + a_2)e'^2 v} < 0 \; *$$

$$(4.15) \qquad \frac{dw}{dT} = e'v \frac{dE}{dT} < 0$$

So education falls as land size increases. This makes sense if labor falls with increased wealth--why spend more on education? (Why spend anything for education in the large landlord model in Section 1.8, if required supervision and hence total labor = 0?)

Employee Education:

Employee education can be modelled by modifying the large landlord model of Sec. 1.8, with $k = 0$, according to a couple of assumptions:

a. Education of employees proportionally increases the wage paid:

$$\text{Max:} \quad P = f(T-E,eH) - veH$$

This model blows up at constant returns to scale, since the denominator of the partial derivatives equals $e^2(f_{11}f_{22} - (f_{12})^2)$.

b. Education of employees does not affect the wage paid:

$$\text{Max:} \quad P = f(T-E,eH) - vH$$

In this case, education of employees falls with increasing land size.

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*An apparent problem arises with the denominator of the above expressions: $e''L + (a_1Z+a_2)e'^2v = e''L + (a_1Z+a_2)f_1{}^2/L^2v$. For as L falls, it looks like the sign must eventually change from negative to positive, at which point the expressions "blow up". However, the explosion depends on assuming that (a_1Z+a_2) remains > 0. This is a reasonable assumption to rule out backwards bending labor supply curves--that the price effect of a wage increase outweighs the income effect. But here, income increases while wage falls, so the assumption need no longer hold.

2.5 Performance and Transactions Costs[C]

Return to the large landlord of Sec. 1.8, the one who only supervises hired labor at a rate, k. But now assume k is not fixed. Rather, the more the landlord supervises, the better his employees perform. Better performance may mean they work faster. It may also mean they bungle less often, so the variance in quality of output falls.

However, assume diminishing returns to supervision. So if $m(k)$ is performance as a function of supervision rate, k, then $m' > 0$ and $m'' < 0$. Also assume $m - m'k \geq 0$, --true if $m(0) \geq 0$, which means the level of performance is zero or better at a zero supervision rate. $m(k)H$ is the effective hired labor supply.

Then the richer the landlord, the lower his level of supervision, k, and hence the lower the performance of his employees, $m(k)$. Otherwise, the results of the large landlord model, Sec. 1.8, remain unaltered.

The Maximization Problem:

The landlord's firm maximizes profit, which now depends on the endogenous level of supervision k, as well as hired labor, H:

$$(5.1) \qquad P = f(T,mH) - vH - wkH$$

obtaining the first-order conditions:

$$(5.2) \qquad mf_2 - v - kw = 0$$

$$(5.3) \qquad m'f_2 - w = 0$$

The effective marginal product of labor equals the landlord and employee's combined wage (5.2). The change in the effective marginal product with supervision rate equals the landlord's wage (5.3).

The equilibrium conditions, together with the landlord's labor supply equation ($L = kH = a(P+wD,w)$), implicitly give supervision rate and other endogenous variables as a function of land size. Derivatives show the effect of increased land size:

Partial Derivatives with Respect to Land Size, T:

$$(5.4) \qquad J = -k^2m''f_2 - Hf_{22}(m-m'k)^2 + m^2f_{22}m'f_2(a_1Z+a_2) \quad > 0$$

$$(5.5) \qquad \frac{dk}{dT} = -\frac{(m-m'k)\,(kf_{12} + a_1f_1mf_{22})}{J} \quad < 0$$

$$(5.6) \qquad \frac{dw}{dT} = -\frac{mm''f_2\,(kf_{12} + a_1f_1mf_{22})}{J} \quad > 0$$

$$(5.7) \qquad \frac{dL}{dT} = a_1f_1 + (a_1Z+a_2)\frac{dw}{dT} \quad > 0 \quad \text{by assumption that wage effect dominates}$$

$$(5.8) \qquad \frac{dH}{dT} = \frac{1}{k}\left[\frac{dL}{dT} - H\frac{dk}{dT}\right] \quad > 0$$

$$(5.9) \qquad \frac{d}{dT}(mH) = \frac{m}{k}\left[\frac{dL}{dT} - H\left(1-\frac{m'k}{m}\right)\frac{dk}{dT}\right] \quad > 0$$

$$(5.10) \qquad \frac{d}{dT}(f_2) = -\frac{m''kf_2\,(kf_{12} + a_1f_1mf_{22})}{J} = \frac{k}{m}\frac{dw}{dT} \quad > 0$$

Obviously, all the other results of the large landlord model in Sec. 1.8 continue to hold.

2.6 Skill and Wage of Employees, With and Without Transactions Costs[C]

Again, return to the large landlord of Sec. 1.8.

Now suppose that hired labor comes in a spectrum of skill levels. Skill here means essentially the same as performance in the last section: more skilled workers may do the job faster, with fewer bungles. To the spectrum of skill corresponds a spectrum of wages, as more skilled workers command higher pay. In addition, all else being equal, better pay motivates workers to demonstrate more skill. So skill can be written as a function of the wage, v, paid hired labor: s(v). (Skill also implicitly depends on the exogenous equilibrium wage level in the economy, v_e, determined by the supply and demand of different size landowners; thus $s(v) = s^*(v,v_e)$).

However, assume diminishing returns to skill as a function of wage. That is, the higher the pay rate and skill, the smaller the increase in skill a unit pay increase elicits. So $s(v)$, $s'(v) \geq 0$, but $s''(v) < 0$. Assume also that even persons with zero skill will not work for less than some positive minimum wage v_m. So $s(v_m) = 0$, and $s - s'v < 0$ up to a critical value v_c, but $s - s'v > 0$ beyond v_c.

Assume that skill, like performance, multiplies the effective hired labor supply, sH.

Then the more land the large landowner has, the greater the pay and skill of his employees. The other results in Sec. 1.8 do not change. However, at a zero supervision rate, the pay and skill remain constant such that $s = s'v_c$, regardless of land size.

The Maximization Problem:

The landlord's firm maximizes profit with respect to hired labor H, and rate of pay, v (with supervision rate k again assumed exogenous):

(6.1) $P = f(T,sH) - vH - wkH$

obtaining first-order conditions:

(6.2) $sf_2 - v - kw = 0$

(6.3) $s'f_2 - 1 = 0$

These equations immediately show that, if $k = 0$, then

(6.4) $s - s'v_c = 0$

So with a zero supervision rate, the wage and skill level remain fixed at v_c, independent of land size.

The first-order conditions, together with the equation for the landlord's labor supply ($L = kH = a(P+wD,w)$, implicitly give wage and skill and other variables as a function of land size. Derivatives show the effect of increasing land size:

Partial Derivatives with Respect to Land Size, T:

(6.5) $J = -k^2(s''f_2 + s'^2Hf_{22}) + (a_1Z+a_2)s^2f^2s''f_{22}$

(6.6) $\frac{dv}{dT} = \frac{ks'(kf_{12} + a_1f_1sf_{22})}{J} > 0$

(6.7) $\frac{dw}{dT} = -\frac{sf_2s''(kf_{12} + a_1f_1sf_{22})}{J} > 0$

(6.8) $\frac{dL}{dT} = a_1f_1 + (a_1Z+a_2)\frac{dw}{dT} > 0$ by assumption

(6.9) $\frac{dH}{dT} = \frac{1}{k}\frac{dL}{dT} > 0$

$$(6.10) \qquad \frac{d}{dT}(sH) = s'H\frac{dv}{dT} + s\frac{dH}{dT} > 0$$

$$(6.11) \qquad \frac{d}{dT}(f_2) = -\frac{s''}{s'^2}\frac{dv}{dT} > 0$$

Obviously, the other results of the large landlord model continue to hold.

2.7 Rental and Leverage, With and Without Transactions Costs[C]

(A) Rental and Leverage Without Transactions Costs:

Absent transactions costs, assume production is linear homogeneous. Then there must be one fixed marketwide rate of rental for land, h, which equals f_1, the marginal product of land everywhere.

Suppose there are no transactions costs in the rental market, but transactions costs implicitly keep farmers from hiring labor or being hired. (This is the inverse of the peasant model in Sec. 1.6, which permits hiring but not renting). So a Clone tenant farmer can rent additional land, V, at the fixed rate, h per acre. He operates a quantity of land T + V, and pays rental hV. His output $F = F(T+V,L)$, while his "financial leverage" $NG = F/(F - hV)$.

As the size of the land he owns increases, his wage remains constant, and so his labor supply falls.

The more land the farmer owns, the less he will rent. In fact, the more he owns, the less the sum of owned and rented land. <u>So the richer the farmer, the smaller his farm!</u>

Financial leverage--the ratio of gross income to gross income less rental payments--declines sharply as land size increases.

Of course, absent any obstacles to renting <u>or</u> hiring, the owner is no longer linked to "his" firm. In fact it becomes impossible to distinguish owned from rented land--as all owners become merely rent-collectors.

(A) The Maximization Problem Without Transactions Costs:

The Clone tenant farmer's firm maximizes profit with respect to L and V:

$$(7.1) \qquad P = f(T+V,L) - wL - hV$$

obtaining first-order conditions:

$$(7.2) \qquad w - f_2 = 0$$

$$(7.3) \qquad h - f_1 = 0$$

Since rent, h, remains constant, then by the assumption of linear homogeneity, wage and marginal product of labor remain constant.

The first-order conditions, together with the landowner's labor supply equation ($L = a(P+wD,w)$), implicitly give rented land and other variables as a function of land size. Derivatives show the effect of increased land size:

<u>Partial Derivatives with Respect to Land Size, T:</u>

$$(7.4) \qquad \frac{dL}{dT} = a_1 f_1 < 0 \qquad \text{labor supply falls}$$

$$(7.5) \qquad \frac{dV}{dT} = -\left[1 + \frac{f_{12}}{f_{11}} a_1 f_1\right] < 0 \qquad \text{rented land falls}$$

$$(7.6) \qquad \frac{d}{dT}(T+V) = -\frac{f_{12}}{f_{11}} a_1 f_1 < 0 \qquad \text{total land operated falls}$$

$$(7.7) \qquad \frac{dF}{dT} = h \frac{d}{dT}(T+V) + w \frac{dL}{dT} \qquad \text{output falls}$$

$$(7.8) \qquad \frac{d}{dT}\left(\frac{V}{F}\right) = -\frac{1}{F^2}\left[\frac{f_{12}}{f_{11}} a_1 f_1 (F-hV) + (F+hVa_1 w)\right] < 0$$

ratio of rented land to output falls. ($a_1 w > -1$, from 1.4)

$$(7.9) \qquad \frac{d}{dT}(NG) = (NG)^2 h \frac{d}{dT}\left(\frac{V}{F}\right) < 0 \qquad \text{financial leverage falls}$$

(B) Rental and Leverage With Transactions Costs:

Suppose the tenant farmer does not face a fixed rental rate, h. h might vary in two plausible ways, or a combination of the two:

i. h might fall with quantity of land rented, V. For if the lessor of the land enjoys economies of scale in supervision--highly plausible--then per acre rent will fall as supervision costs spread over more acres.

However, if rent per acre simply falls with quantity rented, then the silly results of the model without transactions costs (fixed h) hold *a fortiori*. A richer farmer will operate an even smaller farm! Therefore i. cannot hold in isolation.

ii. h may rise with the ratio of rented to owned land, V/T. This makes sense if the expected collection rate falls with V/T, whether or not the lessor is risk-averse. That is, suppose there is an equilibrium rent level in the economy, h_e. Lessors expect to collect only a fraction, x: $0 < x < 1$, of h, the nominal rent. This fraction depends on both the lessor's level of supervision, k, and the ratio, V/T: $x(k,V/T)$, $x_1 > 0$, $x_2 < 0$. So to make their expected rent collection equal to h_e, lessors must charge a nominal rent $h = h_e/x$. If x falls as V/T rises, for a given level of supervision, then nominal rent must rise. If lessors *are* risk-averse, nominal rent must rise even more than enough to compensate for the decline in expected collection rate.

iii. There may be a combined effect, with ii. dominating. Assume ii. alone holds, for greater simplicity of modelling.

Propositions:

Suppose the rental rate, h, does rise significantly with V/T. Then: The marginal product of land exceeds the rental rate.

Richer farmers rent more instead of less land, and operate larger farms.

Farmers' labor supply, wage, and output rise with wealth.

Unless the rental rate, h, rises extremely rapidly with V/T, financial leverage still falls with wealth. But it does not fall as fast as for constant rental h.

Notation and Assumptions for This Section, with Transactions Costs:

$h(V/T)$ -- per acre rent as a function of V/T

$h' > 0$ rent increases with ratio V/T

$h'' > 0$ or $-h''V/T < 2h'$ (2nd deriv > 0 or dominated by first)

The rental rates also implicitly depends on lessors' supervision rate, k. But k does not appear, since the lessee cannot affect it.

The Maximization Problem With Transactions Costs:

The landowner's firm maximizes profit with respect to L and V:

$$(7.10) \qquad P = f(T+V,L) - wL - h(V/T)V$$

obtaining first-order conditions:

$$(7.11) \qquad w - f_2 = 0$$

$$(7.12) \qquad f_1 - h - h'\frac{V}{T} = 0$$

The second condition shows that the marginal product of land now exceeds the rent.

The first-order conditions, together with the landowner's labor supply equation ($L = a(P+wD,w)$), implicitly give land rented and other variables as a function of land size. Derivatives show the effect of increased land size.

Partial Derivatives with Respect to Land Size, T:

Let: $J_1 = \frac{2h'}{T} + \frac{Vh''}{T^2} > 0$ by assumption about $h(V/T)$

Then:

(7.13) $$J = -f_{11} + [1 - f_{22}(a_1 Z + a_2)]\, J_1 > 0$$

(7.14) $$\frac{dV}{dT} = \frac{1}{J}\Big[\underbrace{a_1 f_{12}\big(f_1 + \frac{h'V^2}{T^2}\big) + f_{11}}_{<\,0} + \underbrace{\big(\frac{V}{T} - f_{22}(a_1 Z + a_2)(1 + \frac{V}{T}) \big) J_1}_{>\,0} \Big]$$

if J_1 is large enough, rented land rises

(7.15) $$\frac{d}{dT}(T+V) = 1 + \frac{dV}{dT}$$ total land rises if $\frac{dV}{dT} > -1$

(7.16) $$\frac{dw}{dT} = \frac{d}{dT}(f_2) = \frac{J_1}{J}\Big[f_{12}(1 + \frac{V}{T}) + f_{22} a_1 \big(f_1 + \frac{h'V^2}{T^2}\big) \Big] > 0$$

wage and MP labor rise

(7.17) $$\frac{dL}{dT} = a_1\big(f_1 + \frac{h'V^2}{T^2}\big) + (a_1 Z + a_2)\frac{dw}{dT} > 0$$ by assumption, if $\frac{dw}{dT} \gg 0$

labor supply increases

(7.18) $$\frac{d}{dT}\Big(\frac{V}{T}\Big) = \frac{1}{JT}\Big[a_1 f_{12}\big(f_1 + \frac{h'V^2}{T^2}\big) + f_{11}(1 + \frac{V}{T}) - J_1 f_{22}(a_1 Z + a_2) \Big]$$

must be < 0, so $\frac{dV}{dT} < \frac{V}{T}$, and ratio of rented to owned land falls,

because:

(7.19) $$\frac{d}{dT}(f_1) = \frac{d}{dT}\big(h + h'\frac{V}{T}\big) = TJ_1 \frac{d}{dT}\Big(\frac{V}{T}\Big) < 0$$

MP land must fall (assuming no great economies of scale) because MP labor rises.

(7.20) $$\frac{dh}{dT} = h' \frac{d}{dT}\left(\frac{V}{T}\right) < 0$$ rental rate falls

(7.21) $$\frac{d}{dT}(f_1 - h) = \frac{d}{dT}\left(\frac{h'V}{T}\right) = \left(h' + h''\frac{V}{T}\right)\frac{d}{dT}\left(\frac{V}{T}\right) < 0 \ ?$$

gap between MP land and rental rate closes?

(7.22) $$\frac{dF}{dT} = f_1\left(1 + \frac{dV}{dT}\right) + f_2 \frac{dL}{dT} > 0$$ if J_1 large enough

output increases

(7.23) $$\frac{d}{dT}\left(\frac{hV}{F}\right) = \frac{1}{F^2}\left[- F \frac{h'V^2}{T^2} - hVf_1 + f_1(F-hV)\frac{dV}{dT} - hVf_2 \frac{dL}{dT}\right]$$

< 0 unless $\frac{dV}{dT}$ very large -- close to $\frac{V}{T}$

(If $\frac{dV}{dT} = \frac{V}{T}$, then:

$$\frac{d}{dT}\left(\frac{hV}{F}\right) = \frac{hVf_2}{F^2}\left[\frac{L}{T} - \frac{dL}{dT}\right] > 0 \ )$$

(7.24) $$\frac{d}{dT}(NG) = (NG)^2 \frac{d}{dT}\left(\frac{hV}{F}\right) < 0$$

So financial leverage may fall even when $\frac{dV}{dT}$ and $\frac{dL}{dT} > 0$.

2.8 Rental and Parcel Size, With Transactions Costs[C]

Suppose a large landowner cannot hire, but can only rent out his land in equal size parcels. So if he owns T acres, and rents out n parcels, each parcel has a size T/n. Transactions costs appear in two forms. First, the landowner must spend a fixed time, K, supervising the lessee of each parcel, making his total labor nK. Second, the rent obtainable per acre falls as parcel size increases, because as shown in 2.7, the lessees of larger parcels are less leveraged, (and conceivably more reliable personally). That is, if rent is $r(T/n)$, $r'(T/n) < 0$. However, the rate of decrease in rent logically must taper off as parcel size increases so that $r''(T/n) > 0$, and $r + r'T/n > 0$.

(Notice that rent does not depend on the lessee's ratio of rented to owned land, --because the large landowner, the lessor, does not control this variable independent of rent. That is, in choosing a parcel size and rent, he simultaneously chooses his lessee's other characteristics).

Propositions:

A larger landowner rents out more but larger parcels. So the rent obtained per acre falls. The landowner's personal wage rises, and so by assumption does his labor supply.

Due to the fall in rent, and rise in wage, all the other results of the basic landowner model, the "farmer" (Sec. 1.5) continue to hold. That is, output (= rent collected) per acre falls, but output per manhour (average product of labor) rises, and so forth.

Absent transactions costs--ie., no supervision requirement and a fixed rent, parcel size remains completely indeterminate.

The Maximization Problem:

The landowner's profit is the difference between rent obtained for all n parcels, and the landowner's wages for supervising, $wL = wKn$. His firm maximizes profit with respect to n:

$$(8.1) \qquad P = r(T/n)T - wKn$$

obtaining the first-order condition:

$$(8.2) \qquad w + \frac{r'}{k}\left(\frac{T}{n}\right)^2 = 0$$

Substitution from this condition shows that in equilibrium:

$$(8.3) \qquad P = T \times (r + r'T/n)$$

> 0 by the assumption that rent falls at a decreasing rate.

Therefore if:

$$(8.4) \qquad 2r' + r''T/n = \frac{d}{d(T/n)}(r + r'T/n) < 0$$

then profit per acre falls as parcel size increases. Assume this is the case. This assumption means that while rent falls at a decreasing rate as parcel size increases, the rate of decrease does not taper off too quickly.

The first-order condition and the landowner's labor supply equation ($L = kn = a(P+wD,w)$) together implicitly give the number of parcels and other variables as a function of land size. Derivatives show the effect of increased land size:

Partial Derivatives with Respect to Land Size, T:

Let: $\quad J_1 = 2r' + r''T/n < 0$ by assumption in (8.4)

Then:

(8.5) $J = -(a_1 Z + a_2)T^2 J_1 + K^2 n^3 > 0$

(8.6) $\frac{dw}{dT} = -\frac{J_1}{J}(Kn - a_1 P)T^2 > 0$ landowner's wage rises

(8.7) $\frac{dL}{dT} = K\frac{dn}{dT} = \left[a_1 \frac{P}{T} + (a_1 Z + a_2)\frac{dw}{dT} \right] > 0$ labor and number of parcels rise

(8.8) $\frac{d}{dT}\left(\frac{T}{n}\right) = \frac{Kn}{J}(Kn - a_1 P) > 0$ parcel size increases

(8.9) $\frac{dr}{dT} = r' \frac{d}{dT}\left(\frac{T}{n}\right) < 0$ rent = MP land = revenue per acre fall

(8.10) $\frac{d}{dT}(rT) = r + r' \frac{d}{dT}\left(\frac{T}{n}\right) > 0$ gross revenue rises (by assumptions about r)

(8.11) $\frac{d}{dT}\left(\frac{rT}{Kn}\right) = \frac{P}{KT}\frac{d}{dT}\left(\frac{T}{n}\right) > 0$ average product of landlord's labor rises

2.9 Wealth, Natural Ability, and Firm Size[C]

Suppose the large landlord of Sec. 1.8 can also rent land at a rate h(V/T) as in Sec. 2.7. In addition, suppose the natural ability of large landlords varies (exogenously), as in Sec. 2.3.

Then, first, all the propositions about rental of land in Sec. 2.7 B (the model with transactions costs) apply directly to the large landlord. That is, his personal wage, personal labor supply and hired labor supply and output rise with wealth (land size), but leverage falls, and so forth.

Second, a more able landlord of given wealth rents more land. But he hires even more labor. So not only does his output rise with ability, but so does output per acre. And of course his output per manhour (average product of labor) falls as ability increases, reflecting the more intensive use of his land. His leverage increases.

So firm size as measured by acres of land operated rises with natural ability. Size measured by output or employees rises even faster.

Consequently, given a production function and market wage, the area of land a landlord owns and his natural ability together fully determine the size of his firm, --measured by land operated, employees, or output.

The Large Landlord with Rental:

A simple transformation turns the rental model of Sec. 2.7 into a model of a landowner who hires and rents, with transactions costs in both hiring and renting.

The large landlord of Sec. 1.8, supervises his employees at a rate k, so his labor L = kH. Assume he can rent land too, as in Sec. 2.7.

Then his firm maximizes profit:

$$(9.1) \qquad P = f(T+V,H) - vH - wL - hV = f(T+V,L/k) - (v/k + w)L - hV$$

$$= g(T+V,L) - wL - hV$$

where $g(T+V,L) \equiv f(T+V,L/k) - vL/k$.

As shown in Sec. 1.8, the function g behaves just like the function f. So all the results apply from the rental model with transactions costs in Sec. 2.7.

Moreover, maximization of profit determines firm size, measured by land size T + V, or output, or number of employee manhours H = L/k, without any arbitrary assumptions limiting hiring or rental.

Firm Size and Natural Ability:

Now suppose that the large landlord's natural ability, b, varies. His firm maximizes profit:

$$(9.2) \qquad P = g(T+V,bL) - wL - hV$$

obtaining first-order conditions:

$$(9.3) \qquad w - bg_2 = 0 \qquad \text{(analogous to 3.2)}$$

$$(9.4) \qquad g_1 - h - h'\frac{V}{T} \qquad \text{(analogous to 7.12)}$$

The first-order conditions plus the landlord's personal labor supply (L = a(P+wD,w)) together give land rented, labor hired, and other variables as a function of natural ability and land size. Derivatives show what happens as natural ability increases:

Derivatives with Respect to Natural Ability, b:

Let: $\qquad J_1 = \frac{1}{T}(2h' + h''\frac{V}{T}) > 0$ as in Sec. 2.7

(9.5) $$J = -g_{11} + [1 - b^2 g_{22}(a_1 Z + a_2)]J_1$$

(9.6) $$\frac{dV}{db} = \frac{g_{12}}{J}[L(1+a_1 w) + w(a_1 Z + a_2)] \quad > 0 \quad \text{rented land rises}$$

(9.7) $$\frac{dH}{db} = \frac{1}{k}\frac{d}{db}(bL) = \frac{1}{kJ}(-g_{11} + J_1)[L(1+a_1 w) + w(a_1 Z + a_2)] \quad > 0$$

$$= \frac{(-g_{11} + J_1)}{kg_{12}}\frac{dV}{db} = [\frac{H}{T+V} + \frac{J_1}{kg_{12}}]\frac{dV}{db} \quad > \frac{H}{T+V}\frac{dV}{db}$$

so hired labor labor increases, proportionally faster than rented land.

(9.8) $$\frac{d}{db}(f_2) = k\frac{d}{db}(g_2) = \frac{kJ_1 g_{22}}{J}[L(1+a_1 w) + w(a_1 Z + a_2)] \quad < \quad 0$$

marginal product of labor falls.

(9.10) $$\frac{d}{db} f(T+V,H) = f_1 \frac{dV}{db} + f_2 \frac{dH}{db} \quad > \quad 0 \quad \text{output increases}$$

(9.11) $$\frac{d}{db}\frac{hV}{F} = \frac{1}{F^2}\frac{dV}{db}[\, f_1(f_1 T + f_2 H) - hVf_2(\frac{H}{T+V} - \frac{J_1}{g_{12}})\,]$$

$$> \frac{f_1}{F^2}\frac{dV}{db}[\, f_1 T + f_2 H\,(\frac{T}{T+V} - \frac{J_1}{g_{12}})\,] \quad \text{assume} \quad > 0$$

clearly > 0 for small V. Also > 0 for large V, since $f_2 H \to 0$ as ratio of labor to land rises.

(9.12) $$\frac{d}{db}(\,NG\,) = \frac{d}{db}(\,\frac{F}{F - hV}\,) \quad > \quad 0 \quad \text{financial leverage rises}$$

Since the ratio of labor to land increases, output per acre increases, while output per manhour falls.

Since the ratio of rented to owned land rises, rent increases. Marginal product of land, which always exceeds rent, increases too.