A Classical Model of Distribution, Productivity and Growth: 
Adam Smith was Right

BY MARY M. CLEVELAND

The classical economists recognized three factors of production: land, labor and capital. They took the distribution of land ownership as exogenous, a product of history and institutions. However, Adam Smith observed that concentrated land ownership, “the engrossment of land,” hindered productivity and growth. A modern three-factor model shows that, given transactions costs, inequality of land ownership necessarily creates a land, labor and capital market failure; the greater the inequality, the more severe the failure. This failure can account for the low productivity and low growth rates of third world countries, as well as the close connection between distribution of wealth and distribution of income and wages. A three-factor numerical general equilibrium model provides a graphic illustration.

I. Introduction

Adam Smith devotes much of the Wealth of Nations to skewering various parasites on the body economic. He especially targets the “great proprietor,” who leads a life of indolence and luxury on the rents of his estate, whose management he neglects. When land is “engrossed” that is, occupied primarily by great proprietors, productivity and growth suffer. He attributes the prosperity of the British Colonies in North America to the practice of distributing land in small holdings, and blames the backwardness of the Spanish colonies on the engrossment of land. B IV.7.41, p 671. (Adam Smith 2000).

Fast forward to the twenty-first century. The role of institutions in promoting or hindering growth has become a hot topic. For example, Acemoglu, Johnson and Robinson make an argument similar to Smith’s. They ask why the richest European colonies at the end of the 15th century are now the poorest while the poorest colonies are now the richest. They argue that, “a cluster of institutions ensuring secure property rights for a broad cross section of society, which we refer to as institutions of private property, are essential for investment incentives and successful economic performance. In contrast, extractive institutions, which concentrate power in the hands of a small elite and create a high risk of expropriation for the majority of the population, are likely to discourage investment and economic development.” European colonialism created an “institutional reversal”: it “led to the development of institutions of private property in previously poor areas, while introducing extractive institutions or maintaining existing extractive institutions in previously prosperous places.” ((Daron Acemoglu et al. 2002) p 1235).

Meanwhile, growth theorists are abandoning a long-held hypothesis originally proposed by Simon Kuznets (Simon Kuznets 1955), that growth necessarily requires initially greater inequality, as surplus workers are drawn from an inefficient agricultural sector into a modern industrial sector. The explosive development of the Asian “Tigers”—South Korea, Taiwan Singapore, Hong Kong, Japan, Indonesia, Thailand and Malaysia—has unequivocally demonstrated the close association of relative equality and high rates of growth. See Figure 1 on page 9 below.
Consequently, the last decade has seen a burst of models attempting to characterize the relationship between inequality and growth. (Alberto Alesina, Roberto Perotti 1994; Oded Galor, Joseph Zeira 1993; Roberto Chang 1994). In a 1999 JEL review entitled, “Inequality and Economic Growth: the Perspective of the New Growth Theories,” Aghion, Caroli and Garcia-Peñalosa present a growth model of their own, in which agents are endowed with varying amounts of capital, which they can invest subject to diminishing returns. Given capital market failure, richer agents cannot lend to poorer ones, reducing overall output and growth. Hence, a lump-sum redistribution of endowments will promote growth. In the second part of the paper, they review a number of papers on the relationship between growth and income inequality, reaching no conclusions and positing no relationship to capital endowments. (Philippe Aghion et al. 1999).

Unfortunately, the new growth models do not integrate well with the institutional approaches to the problem of inequality and growth. They do not rely on, explain or sometimes even acknowledge the most dramatic characteristic of third world economies: the concentration of land ownership in the hands of a few dominant families, and the relatively low-intensity use of that land compared to smallholdings. Nor do they integrate theories of distribution of wealth with distribution of income and wages, even though the two are obviously closely linked.

I believe that a three-factor model built on land, labor and capital best captures the essence of the institutional view, including that of Adam Smith, and yields the most productive insights on the relationship between inequality, productivity and growth.

II. Land as a Factor of Production Separate from Capital

The classical economists recognize three factors of production: land, labor and capital. Smith and Ricardo regularly illustrate land concepts with farmland and mines. But they clearly include not just territory, but all natural resources subject to “appropriation”—what we would today call “privatization.” (Modern land titles include oil leases, electromagnetic spectrum, pollution rights, grazing rights, rights of way, and taxi medallions.) Land yields rent, which Smith wrote, “is naturally a monopoly price. It is not at all proportioned to what the landlord may have laid out upon the improvement of the land, or to what he can afford to take; but to what the farmer can afford to give.” (p 167). It was left to Ricardo to prove rigorously that the rent of land is determined at the margin. (David Ricardo 1996). Smith and Ricardo view capital as a physical stock, divided in turn into “circulating” capital, like materials and goods in process; and “fixed” capital like tools and buildings. Capital regularly turns over. If it does not yield a profit, it will not be reproduced. Capital includes the so-called “wage fund”: the goods that capitalists effectively “advance” to workers at the beginning of a production cycle, and recover with profit at the end of the cycle.

Mainstream neoclassical economics today recognizes land, if at all, as a variety of durable capital. (However, land is sneaking back into environmental economics in the guise of “natural capital.”) The concept of capital turning over has also largely faded from consideration; capital is more often treated as an abstract fund to be invested.

By merging land with capital, neoclassical economics strips land ownership of its political connotations and its historical origins. In Smith’s day, landowners, the “great proprietors,” were a powerful political class. The very term “appropriation” of land suggests the origin of landed property in conquest, political favors or theft. In the eighteenth and nineteenth centuries, large English landowners “enclosed” previously common land in order to raise sheep, evicting the
tenant farmers, often with the connivance of local officials. Today we use the benign term "privatization" to denote turning over public property to private entities—that is if we notice at all when Congress grants $70 billion in spectrum rights to existing broadcast companies.

Systems of land ownership are institutions, which can either promote or hinder productivity and growth. Except for times of conquest or revolution, such systems persist over centuries. Consequently, the explicit inclusion of land as a distinct factor of production separate from capital ties models more directly to political reality. (Mason Gaffney 1994).

III. The Pervasiveness of Regressive Land Use, and Its Association with Inequality

Adam Smith notes a phenomenon apparent both in 18th century Britain and European colonies: regressive land use. That is, holding land quality constant, larger property owners use their land less intensively than do smaller ones. Smith observes:

To improve land with profit, like all other commercial projects, requires an exact attention to small savings and small gains, of which a man born to great fortune, even though naturally frugal, is very seldom capable... He embellishes perhaps four or five hundred acres in the neighborhood of his house, at ten times the expense which the land is worth after all his improvements; and finds that if he was to improve his whole estate in the same manner, and he has little taste for any other, he would be a bankrupt before he has finished the tenth part of it... (p 416)

A small proprietor, however, who knows every part of his little territory, who views it with all the affection which property, especially small property, naturally inspires, and who upon that account takes pleasure not only in cultivating but in adorning it, is generally of all improvers the most industrious, the most intelligent, and the most successful. (p 448).

Today, regressive land use remains a conspicuous feature of less-developed agriculture, and a still-significant feature of developed agriculture (R. Albert Berry, William R.Cline 1979). Data from a major survey of Latin American land ownership dramatizes the contrast between large and small land holdings. See Figures 2-4, p 10, from (Ernest Feder 1971). Figure 5, p 11, shows how less-developed countries with such land use patterns show dramatically lower output per man-hour and per hectare than do developed countries (Yujiro Hayami, Vernon W.Ruttan 1971).

Development economists take an institutional view. Many argue that growth cannot occur without significant redistribution of land. Irma Adelman in fact insists that redistribution must precede growth (Irma Adelman, Sherman Robinson 1989) p 984. She and other development economists advocate redistribution by land reform—breaking large holdings into small parcels, by land taxes, or by some combination of both (Alain de Janvry 1993). The two most thorough land reforms occurred in Japan just after World War II and in Taiwan in the early '50's, both, by no coincidence, under occupying armies. The reform in Taiwan has been maintained by land taxes. More limited land reforms, as in South Korea, Mexico, Bolivia, and Egypt have also yielded good results (Peter Dorner 1972). But most land reform attempts have founndered on the political power of large landowners. (Alain de Janvry, Elizabeth Sadoulet 1993). For example, the Brazilian 1964 Land Statute provides for taxing the unused land of large landowners, but the law has proved unenforceable. Groups of peasants periodically "invade" and attempt to cultivate large vacant areas—only to be slaughtered by the private armies of the landowners (Lee Alston et al. 1996).
IV. Distribution of Wealth and Distribution of Income

A convincing model of the relationship between distribution of wealth and economic performance must account for the fact that distribution of wealth and distribution of income and wages move together, both in cross-section and over time.

Over the last twenty years, distribution of wealth and income have become increasingly unequal in the United States, as well as in many (but not all) OECD countries. According to Current Population Reports of the US Census, in 1997 the top 1% of income receivers took in 16.6% of income; while in 1998 the top 1% of wealth holders owned 38.1% of net worth and 47.3% of financial wealth. The top 20% received 56.2% of income, and held 83.4% of net worth, and 90.9% of financial wealth. It is not just inequality of income that has increased—which might be expected, since income includes property income in the upper brackets—but also inequality of wages has increased. Over a longer time span, distribution of wealth and income clearly move with the business cycle. Inequality hit a peak in 1929, with the top 1% of wealthholders owning 44.2% of net worth, reached a low of 19.9% in 1976, and as of 1998, had risen again to 38.1% (Edward N. Wolff 2000).

Many explanations have been offered for the recent trend to greater inequality. These include technological bias (Frank Levy 1998), the increase in stock prices relative to housing prices (Edward N. Wolff 2002, p 41) and worsening in worldwide terms of trade for unskilled labor in developed countries (Adrian Wood 1994).

Another line of explanation points to a continuing shift from a more progressive to a more regressive system of taxes and subsidies. In the United States at the state level, there has been a shift from local property taxes to statewide sales taxes as happened in California after Proposition 13 (1978) and more recently in the Michigan shift of school finance from property to sales taxes. The federal income tax itself, personal and corporate, was highly-progressive when established just before World War I. But since then it has accumulated loopholes and exclusions, while the corporate share has shrunk. With the addition of Social Security taxes, the income tax is steadily devolving into a stiff payroll tax. Meanwhile, the growth in the federal government has brought a growth in federal benefits. Contrary to popular impression, the bulk of these benefits go to the well-to-do. Peter Peterson estimated an annual flow of $570.7 billion to the non-poor vs. $109.8 billion to the poor. The average benefit to households with income over $100,000 exceeds that to households with under $10,000 (Peter G. Peterson 1993).

V. Explaining Regressive Land Use with Transactions Costs and Other Market Obstacles

Capital market failure has long offered a partial explanation for regressive land use. At least where land markets exist, capital market failure hinders poor individuals from buying land. As Rainer Schickelke observed many years ago, when banks make loans, “The principle of allocation is collateral security, not marginal productivity... These two principles tend to work at cross purposes: with increasing collateral security, the marginal productivity of capital tends to decline, and vice versa. Instead of allocating capital to where it is scarce, our credit system allocates it to places where it is ample.” (Rainer Schickelke 1943, p 240). Mason Gaffney has extensively investigated the effects of capital market failure on land use (Mason Gaffney 1975). Nancy Birdsall offers capital market failure as a primary explanation for the association of greater inequality with lower growth; that is, in highly unequal economies, parents can neither
pay for education for their children nor forgo their children’s earnings where education is free, even though such education offers a very high return on investment. (Nancy Birdsall et al. 1995)

But capital market failure cannot in isolation explain regressive land use, nor the connection between wealth inequality and wage inequality. Even if capital markets fail, what keeps rich landlords from hiring poor workers at a wage and intensity such that the capital market constraint does not bind, and productivity remains unaffected? Or what keeps poor workers from accomplishing the same by renting land?

The answer of course is transactions costs, notably supervision costs, now very familiar from the principal-agent literature. An agent has an incentive to shirk. In an uncertain environment, necessarily the case in agriculture, the principal cannot verify the agent’s performance without monitoring him. Supervision costs create a labor market failure that balances land and capital market failure. In fact, capital market failure itself ultimately arises from supervision costs in the capital markets.

A neo-classical general equilibrium model embodying the concept of labor market failure balancing capital market failure has been developed by Mukesh Eswaran and Ashok Kotwal. They set up five agrarian classes depending on working capital, with class boundaries determined by Kuhn-Tucker conditions: pure laborer, laborer-cultivator, self-cultivator, small capitalist and large capitalist. The laborer-cultivator cultivates a small plot and hires labor out, the self-cultivator neither hires in nor out. The small capitalist hires in, works his own land, and supervises his employees. The large capitalist hires in and only supervises his employees. Working capital is partly given, and partly borrowed in proportion to land-ownership, which is exogenous. Eswaran and Kotwal’s model clearly predicts regressive land use. (Mukesh Eswaran, Ashok Kotwal 1986).

Eswaran and Kotwal in turn acknowledge a debt to John Roemer, who in 1982 proposed a similar scheme of five classes dependent on capital ownership. (John Roemer 1982), as well as to Pranab Bardahn, who immediately applied Roemer’s scheme to Indian peasant society (Pranab Bardahn 1982). Roemer’s models are rather complex linear programming exercises involving only labor and capital. Consistent with his Marxist orientation, Roemer dismisses land altogether as a factor of production, by assuming an unlimited supply. This perhaps explains why Eswaran and Kotwal downplay land to focus on working capital, even in their title—“Access to Capital and Agrarian Production Organization.” In fact they conclude that “the creation of institutions capable of accepting as collateral future crops rather than owned land-holdings would prove to be an effective tool for removing poverty as well as for improving efficiency.” (p 196).


Working independently, I constructed models of trade between individuals with different land endowments, given transactions costs. (Mary M. Cleveland 1984). Because these models start with land and labor, and then add capital, I believe they offer a more “realistic” and powerful means to analyze the effect of institutions on inequality and growth.

VI. A General Equilibrium Model of Inequality: a Numerical Illustration

I have constructed a general equilibrium model to illustrate the effects of unequal endowments in a simple economy with transactions costs. It is a one-commodity “agricultural” model similar to Ricardo’s famous corn model, but with all three factors earning their marginal
products. I lay out the underlying mathematics in Appendix A. I used the GAMS optimization program to compute the equilibrium over the range of distributions. Figures 6-21 on pages 12 through 17 show the results.

Assume an economy consisting of 100 identical “farmers” occupying a uniform area of land. The individuals are divided into five groups, consisting of 5, 10, 15, 20, and 50 farmers respectively. Suppose we vary the groups’ share of land varies according to a formula from complete equality to the top 5% having almost 60% of the land. However, we impose a minimum parcel size, so that as we increase the inequality of endowments, an increasing fraction of the bottom 50% become “landless.” (Figures 6 & 7, p 12) As inequality increases, farmers with larger holdings can gain by hiring the labor of farmers with smaller holdings, (or none). However, farmers who hire in labor face a supervision cost in that the “effectiveness” of hired labor is reduced by a factor proportional to the ratio of own labor to hired labor. (Figure 8, p 13).

As inequality increases, the larger farmers produce an increasing share of output. However, overall output falls, due to the supervision requirement. (Figure 9, p 13). Income per capita rises for the richer farmers but falls for the poorer ones. (Figure 10, p 13). Income share also rises for richer farmers but falls for poorer ones; however total income for the economy also falls. (Figure 11, p 14). Total profit (output minus labor costs) falls dramatically over the range; however the profit of the top 5% does not increase dramatically, due to their rising labor costs—including the cost of their own supervisory labor. (Figure 12, p 14).

As inequality increases, the top 5% works longer and longer hours, driven by the rising marginal product of their supervisory labor. (The model does not allow backward-bending labor supply curves. No leisure class here!) The next 10% stays about the same. The labor of the other groups falls, except, dramatically, the labor of the landless. (Figure 13, p 14).

The model distinguishes labor supplied, including supervisory labor, from labor applied to land. As inequality increases labor applied becomes less and less overall and in proportion to labor applied to land, explaining the drop in output. Hence, total labor applied to land falls. (Figure 14, p 15). However, labor hired in by the richer farmers, must always equal labor hired out by the poor farmers or landless. (Figure 15, p 15). As seen, the ratio of labor to land falls as inequality increases. However, the ratio falls dramatically on the land of the top 5%, and a little on the next 10%, while it rises on the land of the smaller farmers. (Figure 16, p 15) Here of course is the explanation of regressive land use.

For every level of inequality, a market wage clears the hired labor market. However, the larger farmers, who hire and supervise the labor of poorer farmers must impute to themselves a wage higher than that market wage. Hence, as inequality increases, the wages of the different groups diverge dramatically. The wage of the top 5% increases almost six-fold over the wage at equality. The wage of the next 10% almost doubles. The wages of the others fall; the wage of the landless laborers, which is the market wage for hired labor, falls to under one fifth of the equality wage. (Figure 17, p 16). Thus inequality in wealth causes not only inequality in income, but wage inequality.

Marginal product of labor on land also diverges as inequality increases, but not so dramatically as wage inequality. That is due to the “wedge” of supervision costs between the wage and the marginal product of labor. (Figure 18, p 16). Marginal product of land also diverges, in the opposite direction. Marginal product of land falls dramatically on the land of the top 5%. (Figure 19, p 16). Since the marginal product of land equals the rent, the imputed rent of
land rises for the smaller farmers, and falls for the largest. This makes sense, as it reflects the increasing scarcity of land for small farmers versus the increasing abundance of land for the large farmers.

And now for a truly paradoxical result: The underlying production function (see Appendix A) is assumed to show economies of scale. As a consequence, the labor share of output plus the land share of output exceeds total output! But how can shares exceed output? Actually, there’s a problem only if the shares go to different individuals. Here, the labor share of output consists of a large share going to the farmers as landowners, and a small share going to small farmers and landless as hired-out labor. Land share going to landowners plus the hired labor share going to different individuals do not exceed output. Landowners receive their land and labor income in a lump, a joint product which they cannot meaningfully split. Supervision costs, which tie the owner’s labor to his land, together with minimum parcel size, keep economies of scale in production from blowing up the economy. (Figures 20 and 21, p 17).

This model is single period economy, with two factors, land and labor. Capital can easily be added, by assuming that labor at the beginning of a period results in production at the end of the period. Capital then is the famous classical “wage fund” advanced to feed labor. If the economy is in a no-growth equilibrium, the results of the single-period model remain unaffected. However, one additional piece of information emerges: as inequality increases, the effective interest rate falls for the top 5% and for the entire economy. (See Appendix B.)

The model is also not a growth model, though it can be converted to one by making assumptions such as those in the model of Aghion et al, that the higher the production of one period the higher the technical production capacity of the next. But it would seem obvious that if inequality reduces the productivity of an economy in the manner described, it also reduces the rate of growth.

VII. Some Implications

The classical-style three-factor model I have presented may seem archaic in light of modern traditions of two-factor economics. However, it offers a powerful approach to a uniform theory of distribution, productivity and growth. By starting with distribution of land ownership, broadly conceived as property rights over natural resources, it recognizes the political-institutional underpinning of distribution. It incorporates transactions costs, normally omitted from macro or growth models. It links distribution securely to economic productivity and growth, land use patterns and distribution of income.

The model offers insights on the recent US history of boom and bust. The boom since the 1970’s has been accompanied by growing wealth and wage inequality. The model says the two are linked. Curiously, it even supports a “technology” explanation for the growing wage gap. Not the conventional explanation that technology has an independent bias towards more capital and skill-intensive methods, but the reverse: greater inequality biases the whole economy towards greater land and capital intensity and against labor, a bias reflected in technology.

The model also suggests appropriate policy on taxation and economic stimulus. Clearly if inequality and transactions costs impede productivity and growth, then tax and stimulus policies should not further inequality or add to transactions costs. Sales taxes do both, in spades. Income taxes less so. Property taxes, well administered, are wealth taxes, intrinsically highly progressive and relatively low on transactions costs (short of bribing the assessor).
On the other hand, well-designed redistributive policies enhance productivity and growth. In explaining the success of the Asian “tigers,” Nancy Birdsall posits a “virtuous cycle” in which egalitarian policies, notably universal access to good education, lead to greater labor productivity and higher wages, which in turn leads to more demand for investment in education, greater savings and investment by lower income groups, harder work by the same groups due to improved opportunity, greater economic stability, greater democracy, and less public spending on favored groups (Nancy Birdsall et al. 1995).

What policies does Adam Smith recommend? First, he says, get rid of laws, like entail and primogeniture, that make it hard to break up property or move it into competent hands. Second, he says, commerce is the enemy of the great proprietors. Get rid of laws, taxes and subsidies that hinder competition. Commerce will thrive, and the great proprietors, given a vast new array of luxuries, will spend themselves into bankruptcy! Meanwhile, provide a basic education to the common people. As to taxation, Smith says, tax the great proprietors income—rent:

> Both ground-rents and the ordinary rent of land are a species of revenue which the owner, in many cases, enjoys without any care or attention of his own. Though a part of this revenue should be taken from him in order to defray the expences of the state, no discouragement will thereby be given to any sort of industry. The annual produce of the land and labour of the society, the real wealth and revenue of the great body of the people, might be the same after such a tax as before. Ground-rents and the ordinary rent of land are, therefore, perhaps, the species of revenue which can best bear to have a peculiar tax imposed upon them. (B V Ch 2 p 909)

The three-factor model developed in this paper permits the taxation of rent. In fact, in a similar model, I have shown the superiority of land to output taxes. (Mary M. Cleveland 1995) In contrast, modern neoclassical models, which merge land with capital, perforce assume any ordinary form of taxation imposes disincentives.

But where is the rent in today’s economy? Actually, it’s there all right, lumped in with profits, masquerading as the “capital gains” of productive investment. Because our tax laws permit a stepped-up basis at death, large estates consist almost entirely of many years’ unrealized capital gains. Yet President Bush has targeted the estate tax for elimination as an unfair “death tax.” Now he proposes to untax dividends, again, untaxing economic rent. Adam Smith’s great proprietors would have felt quite at home.
Figure 1. Income Inequality and Growth of GDP, 1965-89

Note: Income inequality is measured as the ratio of the income shares of the richest 20 percent to the poorest 40 percent of the population. For the East Asian economies, the change in that ratio is shown using the earliest and the latest year for which that ratio is available. For all other economies, the average of that ratio is taken using all years in the period 1965-89 in which the ratio was available. Source: World Bank (1994).

(Nancy Birdsall et al. 1995), p 480.
LAND, LABOR AND OUTPUT IN SOUTH AMERICA

Figure 2

Most of the farm land in South America is held in small or large estates, known as latifundia. Mini-farms (too small to support a family) occupy under 2% of farm land, often of low quality. Family farms occupy another 16%.

Figure 3

The ratio of workers to land is much higher on mini farms than on family farms or small estates. The latter in turn have much higher ratios than the larger estates. The smaller farms cultivate a larger portion of their land (55% on average) while the larger farms run more livestock.

Figure 4

Regressive land use is particularly dramatic in Latin America, and fairly conspicuous in other less-developed countries. However the pattern occurs everywhere.
Figure 5: The Agricultural Productivity Gap Among Countries

International comparison of agricultural output per male worker and per hectare of agricultural land. Output data are 1957-62 averages; and labor and land data are of year closest to 1960.

Note that densely-populated countries like Japan show a high output per hectare, but a relatively low output per man-hour; lightly populated countries like the United States show a high output per man-hour, but low output per hectare. Intermediate countries like Denmark lie in-between. But less-developed countries huddle in the lower left corner with low output per man-hour and per hectare, regardless of population density.

(Yujiro Hayami, Vernon W. Ruttan 1971) p. 71. (The 1985 revised edition contains the same figure with more recent data—graphed on a log scale which obscures the contrast between less-developed and developed countries.)
INCREASING INEQUALITY IN A ONE-PERIOD MODEL

**Figure 6**

Due to minimum parcel size, more and more of the bottom 50% become landless as distribution becomes more unequal. At the limit, 42 of the bottom 50 are landless and 8 are marginal.

**Figure 7**

At the limit, the top 5% hold 56.3% of the land. Their parcel size is 2.25 units versus .056 units for the marginal farmers, a ratio of 40 to 1.
**Figure 8**

The bottom two groups do not hire labor over most of the range.

**Figure 9**

Output falls over the range from 1574 to 1182, about 24.9%. At the limit the top 5%, with 56% of the land, produce 42.3% of output.

**Figure 10**

At equality, per capita income is 23.9. At the limit, the top 5% income is 104.5, versus 4.89 for the marginal farmers of the bottom 50%, and 0.83 for the landless. So, with about 40 times the land of the marginal farmers, the top 5% get about 21x the income. They get about 126 times landless income.
**Figure 11**

Total income over the range falls from 2395 to 1515, a 36.7% drop. At the limit, the top 5% get 34.5% of income, while the bottom 50%, including landless, get 4.9%.

**Figure 12**

Total profit over the range falls from 807 to 431, a 46.6% drop. At the limit, the top 5% get 15.4%, while the 8 marginal farmers in the bottom 50% get 4.1%.

**Figure 13**

Maximum labor time per period = 1. At equality, everyone works .48. At the limit, the top 5% work .85, the marginal farmers work .17, and the landless work .83. Since the wage is the same for marginal and landless, the big difference in labor time shows the sensitivity of labor supply to profit at a low wage.
**Figure 14**

Applied labor falls from 48.3 to 30.4, a 37.1% drop. Supplied labor starts at 48.3, rises to maximum of 56.0, and falls again to 54.5. The final ratio of applied to supplied labor is 56%, which means the other 44% is lost to supervision costs.

**Figure 15**

Maximum hired labor (in or out) is 35. This is about 64% of total supplied labor.

**Figure 16**

The maximum possible ratio is a bit over 4, where MP labor goes to 0. At equality, the ratio is 2.4. At the limit, the top 5% has a ratio of only .663. The marginal farmers have a ratio of 3.49, while the next 20% have the slightly higher ratio of 3.54 -- a reflection of economies of scale at small size.
**Figure 17**

At equality, everyone's wage is 15.9. At the limit, the top 5%'s wage has risen to 91.3; the bottom and market wage has fallen to 2.83.

**Figure 18**

At equality everyone's MP labor = wage = 15.9. At the limit, MP labor for the top 5% rises to 58.1, while that of the marginal farmers falls to 2.83, also their wage and the market wage. Wage and MP labor diverge for farmers who hire in, because MP labor is a weighted average of the farmer's own wage and the market wage.

**Figure 19**

At equality, the marginal product of everyone's land is 66.6. At the limit, MP land for the top 5% falls to 7.8, that of the marginal farmers rises to 69.5, and that of the next 20% to 82.3, a reflection of economies of scale at small land size.
**Figure 20**

Because of economies of scale in production, the sum of labor and land shares exceeds output. The excess falls from 33% of output at equality, to 19.7% at the limit, as supervision costs cut productivity.

**Figure 21**

The sum of hired labor and land shares is always less than total output.
Appendix A

A Single-Period General Equilibrium Model

A. The Consumer-Laborer

The model economy is populated by consumer-laborers. Consumer-laborers who own land are farmers. Those who don’t own land are landless laborers.

A consumer-laborer consumes food and leisure. His labor supply equals the maximum time available in a period minus leisure.

Notation 1--Consumer-Laborer

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$w$</td>
<td>Consumer-laborer’s wage</td>
</tr>
<tr>
<td>$F$</td>
<td>&quot;Food,&quot; assumed to have unit price</td>
</tr>
<tr>
<td>$P$</td>
<td>Profit of Consumer-Laborer (exogenous)</td>
</tr>
<tr>
<td>$D$</td>
<td>Maximum time available for labor in a period, eg 24 hours in a Day.</td>
</tr>
<tr>
<td>$Z$</td>
<td>Leisure of Consumer-Laborer</td>
</tr>
<tr>
<td>$L = D - Z$</td>
<td>Labor supply of Consumer-Laborer</td>
</tr>
<tr>
<td>$u(F, Z) = u(F, D - L)$</td>
<td>Utility function in food and leisure</td>
</tr>
</tbody>
</table>

The consumer-laborer maximizes utility:

(1) \[
\text{Max: } u(F, D-L) \quad \text{st } F = P + wL
\]

First-order conditions:

\[
\frac{u_F}{u_F} - w \geq 0 \quad \frac{u_Z}{u_F} - w \left[ D - Z \right] = 0
\]

In the period of Consumer-Laborer:

(3) \[
y = P + wD
\]

Labor can be expressed as a function of income and wage, or profit and wage, where profit here is exogenous. Assume the labor supply function approaches a limit as wage increases, holding income constant, or as income decreases, holding wage constant:

(4) \[
L = a(y, w) = a(P + wD, w) \quad \text{a}_y < 0; \text{a}_w > 0; \text{a}_{yw} > 0; \text{a}_w < 0; \text{a}_{yw} < 0
\]

Assume further:
\[
\frac{\partial L}{\partial w} = \frac{\partial}{\partial w} a(P + wD, w) = \left[ a_y D + a_w \right] > 0
\]

That is, holding profit constant, labor supply does not bend backward. In general, assume that wage terms, \( aw \), dominate income terms, \( ay \), since any results that hold without backward-bending hold a fortiori with it.

**B. The Consumer-Laborer as a Farmer**

Combine a consumer-laborer with a piece of land to which he applies his labor, making him a farmer. How does farm size affect the farmer’s behavior?

<table>
<thead>
<tr>
<th><strong>Notation 2</strong>---The production function:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T ) ... Size of a parcel of land</td>
</tr>
<tr>
<td>( A ) ... Labor Applied to land parcel</td>
</tr>
<tr>
<td>( F = f(T, A) ) ... Food output from ( T ) ... ( f_T &gt; 0; f_A &gt; 0; f_TA &gt; 0; f_{TT} &lt; 0; f_{AA} &lt; 0 )</td>
</tr>
<tr>
<td>( f_{TT} \cdot f_{AA} - [f_{TA}]^2 &lt; 0 ) and ( f_T f_A T + f_A A &lt; 0 ) for small ( T ), ie, ( \exists ) economies of scale.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Notation 3</strong>---Hiring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumptions for Farmers who Hire In or Out</td>
</tr>
<tr>
<td>( v ) ... Market wage</td>
</tr>
<tr>
<td>( \bar{H} ) ... Hired - out labor</td>
</tr>
<tr>
<td>( \tilde{H} ) ... Hired - in labor</td>
</tr>
<tr>
<td>( e \left( \frac{\bar{H}}{L} \right) = ) Effectiveness of hired labor. ( e' &lt; 0, e'' &gt; 0, e &lt; 1 ) at ( \bar{H} = 0 )</td>
</tr>
<tr>
<td>( e \left( \frac{\bar{H}}{L} \right) \cdot \bar{H} ) ... Effective hired labor supply increases at a decreasing rate with ( \bar{H} ),</td>
</tr>
<tr>
<td>( \Rightarrow e + e' \cdot \frac{\bar{H}}{L} &gt; 0 ) but steadily declining ( \Rightarrow 2 e' + e'' \cdot \frac{\bar{H}}{L} &lt; 0 )</td>
</tr>
</tbody>
</table>

\( \bar{H} = 0 \)
1. Small Farmer Also Works for Hire

Maximize profit:

\[ P = f(T, A) - w \cdot L + v \cdot \tilde{H} \]

subject to:

\[ \tilde{H} + A = L = a(y, w); \quad \tilde{H}, A \geq 0 \]

\[ y = P + wD \]

First-order conditions:

\[ \tilde{H}: \quad [f_A - v] \cdot \tilde{H} = 0; \quad \tilde{H} \geq 0 \]

\[ A: \quad [f_A - w] \cdot A = 0; \quad A \geq 0 \]

Small farmer also works for hire only when:

\[ f_A = v; \quad \tilde{H} > 0 \quad [f_A - w] \cdot A = 0 \]

The small farmer works for hire only as long as the outside wage equals the marginal product of labor on his own land. The more land the small farmer owns, given a market wage, the more he works on his own land, the less he works for hire, and the less he works in total. This is a pure income effect, since wage is fixed. If wage increases, holding land size constant, wage and income effects pull in opposite directions, but by assumption here, there are no backward-bending labor supply curves.

2. Self-sufficient Farmer

The farmer does not work for hire when:

\[ f_A > v; \quad \tilde{H} = 0 \quad [f_A - w] \cdot L = 0 \]

This is the self-sufficient farmer. The farmer’s wage and marginal product of labor exceed the market wage. So he works only on his own land. The more land he owns, the longer hours he works, the higher his wage and marginal product of labor, and the lower the marginal product of his land. For of course the labor to land ratio falls as land size increases.

3. Farmer Can Hire Additional Labor:

Assume the effectiveness of hired labor is less than that of the farmer’s own labor. Moreover, the effectiveness falls as the ratio of hired to own labor rises, due implicitly to the farmer’s increasing difficulty of supervising.

Maximize profit:

\[ P = f(T, A) - w \cdot L - v \cdot \tilde{H} \]

Subject to applied labor is effective hired labor plus owner’s labor:

\[ A = e\left(\frac{\tilde{H}}{L}\right) \cdot \tilde{H} + L \quad e' < 0, e'' > 0, e < 1 \text{ at } \tilde{H} = 0; \text{ etc as above} \]
\[ L = a(y, w); \quad \tilde{H}, A, L \geq 0 \]
\[ y = P + wD \]

First-order conditions:

\[
\tilde{H}: \quad \left[ f_A \left( e + e' \frac{\tilde{H}}{L} \right) - v \right] \cdot \tilde{H} = 0; \quad \tilde{H} \geq 0
\]

\[
L: \quad \left[ f_A \left( 1 - e' \frac{\tilde{H}^2}{L^2} \right) - w \right] \cdot L = 0; \quad L \geq 0
\]

a. Farmer does not hire additional labor.

\[
\tilde{H}: \quad f_A \cdot e - v < 0; \quad \tilde{H} = 0
\]

\[
L: \quad f_A - w = 0; \quad L > 0
\]

The effective marginal product of hired labor is less than the wage for hired labor, \( v \). So the farmer does not hire in labor. If he does not hire out labor either, as in b. above then:

\[
v < f_A = w < \frac{v}{e} \quad \tilde{H} = 0; \quad \tilde{H} = 0
\]

The market wage for hired labor is less than the marginal product of labor on the farmer’s land, which equals his wage. However, the marginal product of hired in labor is less than the wage, due to the lower effectiveness of hired than own labor.

b. Farmer does hire additional labor.

\[
\tilde{H}: \quad f_A \left( e + e' \frac{\tilde{H}}{L} \right) - v = 0; \quad \tilde{H} > 0
\]

\[
L: \quad f_A \left( 1 - e' \frac{\tilde{H}^2}{L^2} \right) - w = 0; \quad L > 0
\]

From the assumptions about effectiveness of hired labor:

\[
0 < e + e' \frac{\tilde{H}}{L} < 1
\]

\[
1 < 1 - e' \frac{\tilde{H}^2}{L^2}
\]

So it follows that:

\[
v < \frac{v}{e + e' \frac{\tilde{H}}{L}} = f_A = \frac{w}{1 - e' \frac{\tilde{H}^2}{L^2}} < w
\]

The marginal product of labor is greater than the wage for hired labor, but less than the farmer’s own wage.
An increase in the outside wage leads to less hiring of labor, and an increase in the average product of labor. If the quantity of hired labor is small, the farmer’s own labor will increase, to substitute for hired labor. If hired labor is large, the farmer’s own labor will decrease.

C. A One-Period Numerical General Equilibrium

Imagine a uniform area of land populated by a number of consumer-laborer-farmers, differing, if at all, only in the quantity of land each owns. For a general equilibrium to work, the following conditions must hold:

1. The quantity of labor hired in by larger farmers must equal the quantity of labor hired out by smaller farmers and landless laborers, at a uniform market wage rate.

2. All the first order conditions and inequalities must hold for the four possible categories: landless laborers, small farmers who also hire out, self-sufficient farmers, and larger farmers who hire in.

For \( i \) individuals in the economy, the first condition may be written:

\[
(19) \quad \sum_i (\bar{H}_i - \bar{\bar{H}}_i) \cdot \nu = 0
\]

--recognizing, of course, that for any individual, either hired-out labor is 0, or hired-in labor is 0, or both.

A numerical simulation model requires making specific assumptions about numbers of individuals, distribution, and functional forms.

Distribution. Imagine the economy consists of a uniform area of 20 units of land occupied by 100 identical farmers. The farmers are divided into five groups, with 5, 10, 15, 20, and 50 individuals respectively. The share of land held by Group \( N_i \) is:

\[
(20) \quad S_i = \frac{N_i^\alpha}{\sum_i N_i^\alpha}
\]

This simple formula makes it possible to vary distribution by changing only one number: \( \alpha \). Distribution is equal for \( \alpha = 1 \). At \( \alpha = -1.4 \), the top 5% holds close to 60% of the land. The extreme figure is commensurate with Atkinson’s estimate for Great Britain in the 1960’s, where the top 1% of wealth-holders had 33-40%, and the top 5% had 59-64%. [Atkinson, 1975, pp. 289 & 308.] Less-developed countries show much greater inequality.

Minimum Parcel Size and Landless Farmers. Assume a minimum parcel size: .05 land units in this model. Consequently, when the share of the bottom 50 farmers falls below .05 per capita, some of them become landless. This way, the remaining “marginal” farmers retain at least the minimum parcel size. When \( \alpha \) hits -0.4, three farmers become landless; at the extreme \( \alpha \) of -1.4, 42 of the bottom 50 farmers are landless. Figures 1 and 2 show the distribution of population and landownership.

Labor Supply. An individual’s labor supply, \( L \), depends on exogenous profit, \( P \), and wage, \( w \):

\[
(21) \quad L = a(y, w) = a(P + wD, w) \ldots a_y < 0; a_w > 0; a_{yw} > 0; a_{yy} < 0; a_{ww} < 0
\]
A simple function that meets these specifications is:

\[ L = \frac{(D-L)w-P}{P_0 + (D-L)w-P}; \quad 0 \leq L < D \]  

P0 “base profit” and D “day” are constants, assumed to be .1 and 1, respectively. L behaves nicely, rising asymptotically towards the limit, D, as wage increases or profit falls. It never bends backwards. P0 affects curvature; the smaller P0, the faster L rises and then flattens. In order to avoid “stacking the deck” against the output tax, and because real world labor supply functions seem fairly inelastic, I chose a small value for P0. Consequently, labor supply functions in the model operate mostly in the flat range, with very limited marginal effects.

**Production Function.** Production depends on land, T, and applied labor, A. There are economies of scale for small T:

\[ F = f(T,A); \quad f_T > 0; f_A > 0; f_{TA} > 0; f_{TT} > 0; f_{AA} > 0 \]

An unconventional but simple function that meets these specifications is:

\[ F = F_0 T^{\frac{1}{a-b}} \frac{(1-a)A}{T} \]

F0, T0, a and b are positive constants, set equal to 80, .1, .1, and .1 in this model. T0>0 creates economies of scale at small scale; the function becomes linear homogeneous at large scale. b >0 in the numerator means that the marginal product of labor can become 0 and then negative as the ratio of labor to land increases. (This is far more plausible than the assumption built into CES functions that the marginal product of labor remains positive at near infinite ratios of labor to land!) b sets a practical limit to the ratio of labor to land at 0 marginal product of labor:

\[ R_{\text{max}} = \left[ 1 + \frac{a}{b} \right]^{\frac{1}{2}} - 1 \]

For a = b, R_{\text{max}} = 4.142.

T0>0 means that as land size increases from 0, the marginal product of land first rises and then falls. An increasing marginal product of land will blow up a general equilibrium model; consequently, minimum parcel size must exceed the critical land size at which the MP land changes direction. However, the higher the ratio of labor to land, the larger the critical land size. b to the rescue! A safe minimum parcel size can be computed as a function of T0 and the ratio of b to a; the lower the ratio, the higher the minimum parcel size. For b=a as assumed, the safe minimum parcel size computes to about .19 T0, or .019. The actual minimum parcel size used in the model was .05.
Effectiveness of Hired Labor. I assumed that farmers who hire in labor face a supervision cost in that the “effectiveness” of hired labor is reduced by a factor proportional to the ratio of hired-in labor, \( \tilde{H} \) to own labor, \( L_A \). A simple formula for effectiveness of hired labor is:

\[
(26) \quad e = \frac{1}{E + \frac{\tilde{H}}{L_A}}
\]

so that effective hired labor supplied is:

\[
(27) \quad e \tilde{H} = \frac{\tilde{H}}{E + \frac{\tilde{H}}{L_A}}
\]

\( E \) is a constant, which must be >1 to make the effectiveness of hired-in labor always < 1, even at the point where hiring starts. For the model, \( E = 1.2 \).

Hence, when labor is hired in, total effective labor applied to land becomes:

\[
(28) \quad A = e \tilde{H} + L_A = \frac{\tilde{H}}{E + \frac{\tilde{H}}{L_A}} + L_A \left[ 1 + \frac{\tilde{H}}{EL_A + \tilde{H}} \right]
\]

The farmer who hires in must work himself. The more he hires, the harder he must work.

Computing the Numerical General Equilibrium Model. Using these explicit functional forms, I wrote out the single-period equations and fed them to the GAMS optimization program. GAMS computed the equilibrium over the range of distributions and output results to a file readable by spreadsheet programs. I used the Borland Quattro Pro spreadsheet program to organize and graph the data.
Appendix B

A Multiple-Period General Equilibrium Model Including Capital

A. The Consumer-Laborer-Saver/Borrower

The model economy is populated by consumer-laborer-saver/borrowers (CLSB). CLSB's who own land are farmers. Those who don't own land are landless laborers.

A CLSB consumes food and leisure in two (or more) periods. His labor supply equals the maximum time available in a period minus leisure. Whatever he saves is returned with interest in the next period.

<table>
<thead>
<tr>
<th>Notation 1--Consumer-Laborer-Saver/Borrower</th>
</tr>
</thead>
<tbody>
<tr>
<td>w ... Wage (exogenous)</td>
</tr>
<tr>
<td>r ... Interest rate (exogenous)</td>
</tr>
<tr>
<td>$F_0, F_1$ ... &quot;Food.&quot; in this period and future</td>
</tr>
<tr>
<td>$P_0, P_1$ ... Profit in this period and future (exogenous)</td>
</tr>
<tr>
<td>$D$ ... Maximum time available for labor in a period, eg 24 hours in a Day.</td>
</tr>
<tr>
<td>$Z$ ... Leisure</td>
</tr>
<tr>
<td>$L = D - Z$ ... Labor supply</td>
</tr>
<tr>
<td>$S$ ... Saving (may be negative)</td>
</tr>
<tr>
<td>$u(F_0, F_1, Z) = u(F_0, F_1, D - L)$ ... Utility function in food and leisure</td>
</tr>
</tbody>
</table>

The CLSB maximizes utility:

\[
\text{Max: } u(F_0, F_1, D - L) \quad \text{st } F_0 = P_0 + wL - S \\
F_1 = P_1 + (1 + r)S
\]

First-order conditions (simplified):

\[
\begin{align*}
\frac{u}{u_{F_0}} - w &= 0 \\
\frac{u}{u_{F_1}} - (1 + r) &= 0
\end{align*}
\]

These equations can be solved implicitly to find labor supply and saving as a function of the exogenous variables:

\[
\begin{align*}
L &= a(P_0, P_1, w, r) \\
S &= a(P_0, P_1, w, r)
\end{align*}
\]

In Appendix A, I laid out in detail the constraints on the behavior of the labor supply as a function of income and wage, income being in turn a function of exogenous profit, $P$. There's no
need to repeat that exercise here. It is sufficient to assume here that the wage effect dominates the labor supply, and the interest rate effect dominates the savings supply. Thus:

\[ \frac{\partial L}{\partial w} > 0 \]

\[ \frac{\partial S}{\partial r} > 0 \]

**B. The Consumer-Laborer as a Farmer-Investor**

Combine a consumer-laborer-saver/borrower with a piece of land, and we get a farmer-investor. He now invests by applying labor—his own or hired or both—to his land. Next period he gets back a profit. How does farm size affect the farmer’s behavior? To keep it simple, assume the farmer applies only his own labor. He maximizes next period profit:

\[ \text{Max: } P_1 = f(T, L) - w \cdot L(1 + r) \]

First order conditions (simplified):

\[ [f_L - w(1 + r)] \cdot L = 0 \]

\[ S - wL = 0 \]

In brief, under these particular constraints, the farmer's saving and investment just equals his internal wage bill. This wage bill is just the classic wage fund. By implication, \( P_0 > 0 \) or the farmer would starve. In a steady state equilibrium, \( P_0 = P_1 \). Then the model becomes indistinguishable from the single period model except that the marginal product of labor now equals the wage times \((1+r)\).

**C. A Steady-State Multi-Period General Equilibrium**

A steady-state multi-period general equilibrium among farmers of different sizes would look pretty much like the single-period general equilibrium. The difference would be \((1+r)\) term. The effective marginal product of labor, whether of the farmer, or of his hired labor, must equal \((1+r)\) times the wage, \( w \) for the internal wage, and \( v \) for the market wage.

There must obviously be constraints on saving or borrowing, such that borrowers cannot borrow more than they can return in the next period, that is \( F \) (food) must remain greater than zero. In a sophisticated model, an individual’s effective interest rate, \( r \), would rise with the ratio of the amount borrowed to his asset (land) or annual production. But even without such "realism," it is obvious that the most perfect capital market possible would still not cure the effects of an unequal distribution of land, given supervision costs. That in turn suggests that efforts to improve third world credit markets cannot accomplish much absent functioning land markets.

A multi-period model also puts an imputed price on land, even in the absence of a land market, and thus reveals the effect of inequality on interest rates. The price of land should be its marginal product divided by the interest rate, in this case imputed. As inequality increases, the marginal product of land falls on large farms, and rises on small ones. Yet by assumption all the land is identical, which means it should have the same price, imputed if not actual. That indicates
that the imputed discount rate, $r$, falls on large farms and rises on small ones. Since overall output falls for the economy, that means the imputed discount rate falls for the entire economy. In other words, the economy behaves more like the large farmers than the small ones.
Reference List


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